Problem 1 Using a protractor, measure the angles $\angle A$ and $\angle B$ of the triangle below. Find the size of the angle $\angle C$, in degrees, without measuring it.
Problem 2 It takes a grandfather’s clock 30 seconds to chime 6 o’clock. How much time would it take the clock to chime 12?

Clock Arithmetic or a Circle as a Number Line

One way to turn a circle into a number line is to divide it into twelve equal parts. In this case, one step is usually called one hour.

0 coincides with 12. The hour hand moves from 0 to 1, from 1 to 2, ... from 11 to 12 just as it would have on the straight number line. However, 12 equals 0 on this circle, so there it goes
again, from 1 to 2, and so on. We write down the fact that 12 equals 0 as

\[ 12 \equiv 0 \pmod{12} \]  \hspace{1cm} (1)

and read it as *12 is congruent to 0 modulo 12*. The usual “=” sign is reserved for the straight number line; we use “≡” on the circle instead. The *mod 12* symbol tells us that the circle is divided into 12 equal parts, so 12 coincides with 0, 13 – with 1, 14 – with 2, and so on. Or in the new notations,

\[ 13 \equiv 1 \pmod{12}, \quad 14 \equiv 2 \pmod{12}, \ldots, \quad 23 \equiv 11 \pmod{12}, \]

\[ 24 \equiv 12 \equiv 0 \pmod{12}. \]

**Problem 3**

\[ 21 \pmod{12} \equiv \]

\[ 80 \pmod{12} \equiv \]

\[ 9 + 4 \equiv \quad (\pmod{12}) \]

\[ 24 - 2 \equiv \quad (\pmod{12}) \]
Problem 4 An experiment in a biological lab starts at 7:00 AM and runs for 80 hours. What time will it end?

Another standard way to turn a circle into a number line is to divide it into 60 equal parts. Depending on the situation, the unit step is called either a minute or a second.

All the numbers living on this number line are considered modulo 60. In particular, \(60 \equiv 0 \pmod{60}\). There are 60 minutes in an hour.
Problem 5

\[ 72 \pmod{60} \equiv \]

\[ 135 \pmod{60} \equiv \]

\[ 55 + 55 \equiv \pmod{60} \]

\[ 240 - 59 \equiv \pmod{60} \]

Problem 6 What is the time, in hours, minutes, and seconds, on the clock below?
There are 24 hours in a day, so one more standard way to turn a circle into a number line is to divide it into 24 equal parts. The US military use the 24-hour clock. The following is a photograph of the 24-hour clock from the USS (United States Ship) *Mullinnix*, the last “all gun” US Navy destroyer in the Pacific, decommissioned in 1982.\(^1\)

\[\text{USS Mullinnix 24-hour clock}^{2}\]

\(^1\)See its homepage at [http://www.ussmullinnix.org/](http://www.ussmullinnix.org/)
\(^2\)Downloaded from [http://www.ussmullinnix.org/MuxMemorabilia.html](http://www.ussmullinnix.org/MuxMemorabilia.html)
Since 60 ÷ 24 is not a whole number, we can’t use the same marks on the face of a 24-hour clock for minutes and hours (to better see this, please find the minute and hour marks on the face of the USS Mullinnix clock). 60 ÷ 12 = 5, so this inconvenience doesn’t exist for the clocks and watches we are used to. On the other hand, to disambiguate between, say, 1 o’clock night time and 1 o’clock afternoon, we have to use the A.M./P.M. notation not needed in the military. In their language, 1 o’clock P.M. is 13:00 (thirteen hundred) hours, plain and simple.

**Problem 7** What time does the USS Mullinnix clock show?

**Problem 8** What is the time on the clock below?
Suppose that this is the time P.M. How would the military call it?

**Problem 9** On the left, draw the civilian clock showing 1:45. On the right, draw the military clock showing the same time P.M.
In the following problems, we will consider the $mod\ 5$ arithmetic, that of a circle divided into five equal parts.

Problem 10

$7\ (mod\ 5) \equiv$

$100\ (mod\ 5) \equiv$

$-1\ (mod\ 5) \equiv$

$1 - 4 \equiv\ (mod\ 5)$
\[ 3 + 3 \equiv (mod \ 5) \]
\[ 2 \times 3 \equiv (mod \ 5) \]
\[ 1 \div 2 \equiv (mod \ 5) \]
\[ 1 \div 3 \equiv (mod \ 5) \]
\[ 4 \times 4 \equiv (mod \ 5) \]
\[ 1 \div 4 \equiv (mod \ 5) \]