Puzzles and Logic

Based on Laura Givental’s presentation at the 2012 Circle on the Road conference.

A part of wizardry training is developing logic skills. In the following exercises, two students will be presented with three wizard’s hats, one with a green label and two with yellow labels.

The students will be asked to close their eyes. One hat will be hidden, the others will be put on the students’ heads. Then the
students will be asked to open their eyes and to figure out the color of their hat labels by observing that of the other student. A student is not allowed to look at her/his own hat. This kind of cheating will be punished by turning the cheat into a toad!

**Problem 1** The students, Alice and Bob, are given two hats. Alice gets the green-labeled hat, Bob gets a yellow-labeled one. Which of the students will be able to figure out the color of her/his hat? How is she/he going to do that?

**Problem 2** This time, Alice and Bob are both given the yellow-labeled hats. Is there a way for them to figure out the color of their hats’ labels? How can they do it?
As the training progresses, three students will be chosen and presented with five hats, three with yellow dots and two with green. Once again, the students will be asked to close their eyes. Two hats will be hidden, three will be put on the students’ heads. The students will be asked to open their eyes and to figure out the color of their hat labels by observing those of the other students.
Problem 3 The students, Alice, Bob, and Charlie, are given the hats. Alice gets a yellow-labeled hat, Bob and Charlie get the hats with green labels. Which of the students will be able to figure out the color of her/his hat? How is she/he going to do that?

Problem 4 This time, Alice and Bob get yellow-labeled hats while Charlie gets the hat with a green label. Which of the students will be able to figure out the color of her/his hat? How is she/he going to do that?
Problem 5 *Finally, all the three students are given the yellow-labeled hats. Can they figure out the color of their labels? How?*

Once upon a time, in a land far, far away there lived a very beautiful princess, the only daughter of a very evil king. Some day, a handsome and very smart prince from a neighboring kingdom came to pay them a visit. The princess and prince fell in love with each other and asked the king for a permission to marry. The evil king didn’t want his daughter to leave. Instead of blessing the marriage, he ordered to put the prince in jail and to prepare for his execution.

The princess begged the king not to kill the prince and finally he agreed. He told the prisoner, “At her Highness’s request, I will give you a chance. Tomorrow you will be brought to my court. You will have to pull a lot. I will put two pieces of paper in the box. One will read LIFE, the other will read DEATH. Whatever piece you pull out, it will be your destiny.”

The king was a very evil man. He ordered his minister to write DEATH on both pieces of paper. The princess overheard the king’s order found a way to warn the prince.
Problem 6 What should the prince do to survive? 
*Hint: kings do not like public embarrassment.*

The prince’s father, the king of the neighboring country, had his peculiarities, too. He designed the following way of punishing criminals. Convicted lawbreakers were given a choice between two doors. Behind each door, there could be either a hungry tiger or a treasure of gold, but not nothing or both. The king would also post some warnings on the doors and then let the criminals choose.

Problem 7 The king took the prisoner to the doors. There was a sign on each door. The first read, “There is gold in this room and there is a tiger in the other.” The sign on the second door read, “There is gold in one of these rooms and in one of these rooms there is a tiger.” “Are the signs true?” asked the prisoner. “One of them is,” replied the king, “but the other is not. Now, make your choice, buddy!” Which door should the prisoner open? Why?
Problem 8 For the second prisoner, the following signs were put on the doors. Door 1: at least one of these rooms contains gold. Door 2: a tiger is in the other room. “Are the signs true?” asked the prisoner. “They are either both true or both false,” replied the king. Which door should the prisoner choose? Why?

Problem 9 In this case, the king explained that, again, the signs were either both true or both false. Sign 1: either this room contains a tiger, or there is gold in the other room. Sign 2: there is gold in the other room. Does the first room contain gold or a tiger? What about the other room?