1) Prove that the distance between any two points inside a triangle is not greater than half the perimeter of the triangle.

2) Given points $A$ and $B$ are on one side of line $l$. We want to draw a path from $A$ to $B$ that intersects $l$. Find the shortest such a path.
3) Point $A$, inside an acute angle, is reflected in either side of the angle to obtain points $B$ and $C$. Line segment $BC$ intersects the sides of the angle at $D$ and $E$ (i.e. $E$ is close to $C$ than $B$). Show that:

a) $EC = EA$ and $BD = DA$.

b) $DE < BD + EC$.

c) $BC/2 > DE$.

4) A fly sits on one vertex of a wooden cube. What is the shortest path it can follow to the opposite vertex?
5) If point $O$ is inside triangle $ABC$, prove that $AO + OC < AB + BC$.

6) Point $C$ lies inside a given right angle, and points $A$ and $B$ lie on its sides. Prove that the perimeter of triangle $ABC$ is not less than twice the distance $OC$, where $O$ is the vertex of the given right angle.
Challenge 1) Prove that the length of median $AM$ in triangle $ABC$ is not greater than half the sum of sides $AB$ and $AC$.

Challenge 2) Prove that in any triangle, the sum of the lengths of the three medians is not greater than the triangle’s perimeter.

Problems are taken from:
- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes
Warm up 1) Do there exist natural numbers $a$ and $b$ such that $ab(a - b) = 65065$?

Warm up 2) Find a point inside a convex quadrilateral such that the sum of the distances from the point to the vertices is minimal.