Exploring the City of Descartes-Part II

Junior Circle 04/03/32011

Recall that all the houses in the City of Descartes have addresses consisting of two numbers (e.g., (3,-5)):

- The first number tells you the number of the "column" the house is in. (Column numbers are positive to the right of the y-axis and negative to the left of the y-axis);
- The second number tells you the number of the "row" the house is in. (Row numbers are positive above the x-axis and negative below the x-axis);

1. For a point (x, y) on the plane, the first number is called x-coordinate and the second one is called the y-coordinate.
   For example, for point $P = (-1, 4)$ we have $x = -1$; $y = 4$.

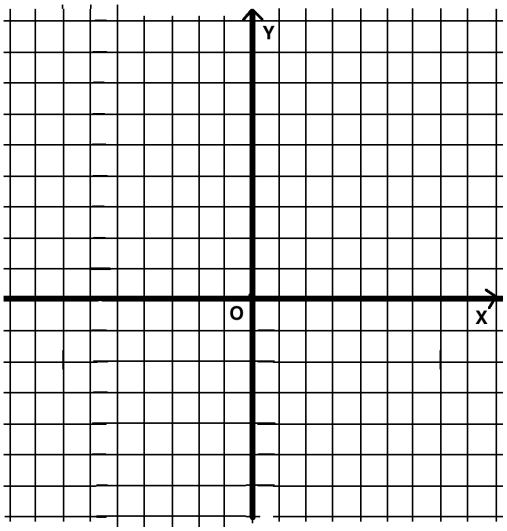
   (a) Find the x-coordinate of the following points:

   i. $A = (3, 2) \implies x =$;
   ii. $B = (4, -5) \implies x =$;
   iii. $C = (-7, -9) \implies x =$;

   (b) Find the y-coordinate of the following points:

   i. $A = (3, 2) \implies y =$;
   ii. $B = (4, -5) \implies y =$;
   iii. $C = (-7, -9) \implies y =$;

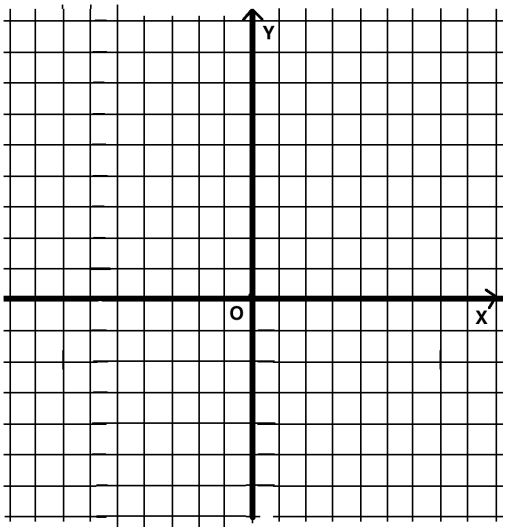
2. Plot several points and reflect them across the x-axis. (Imagine that x-axis is a mirror)



(a) Does the x-coordinate change when you reflect a point across the x-axis? If so, how?

(b) Does the y-coordinate change when you reflect a point across the x-axis? If so, how?

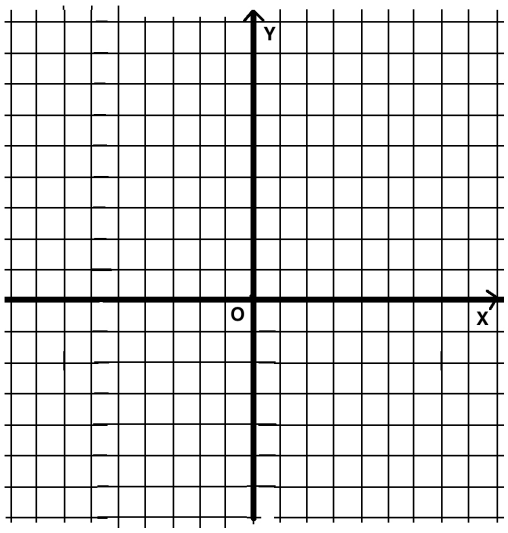
3. Plot several points and reflect them across the y-axis. (Imagine that y-axis is a mirror)



(a) Does the x-coordinate change when you reflect it across the y-axis? If so, how?

(b) Does the y-coordinate change when you reflect it across the y-axis? If so, how?

4. Draw the line $y = x$.



- Reflect the following points across this line and find the coordinates of the reflections:
  (a) $(3, 0) \implies (\quad, \quad)$,
  (b) $(-1, 0) \implies (\quad, \quad)$,
  (c) $(0, 2) \implies (\quad, \quad)$,
  (d) $(0, -3) \implies (\quad, \quad)$,
  (e) $(1, 2) \implies (\quad, \quad)$,
  (f) $(1, 1) \implies (\quad, \quad)$,
  (g) $(3, 2) \implies (\quad, \quad)$,
  (h) $(-4, -1) \implies (\quad, \quad)$.
- How is the address (coordinates) of the reflection related to the address of the given point? Explain.

5. The houses of Amy, Ben, Cindy and Dan are vertices of a square:

   - The center of this square is at the point $O = (0, 0)$;
   - The length of each of the sides of this square equals to 4;
   - Amy's house is directly to the north from Dan's house;
   - Ben's house is east from Amy's house;

   Denote the vertices of this square by A, B, C, D. Mark the houses (vertices) on the picture below and find their addresses:

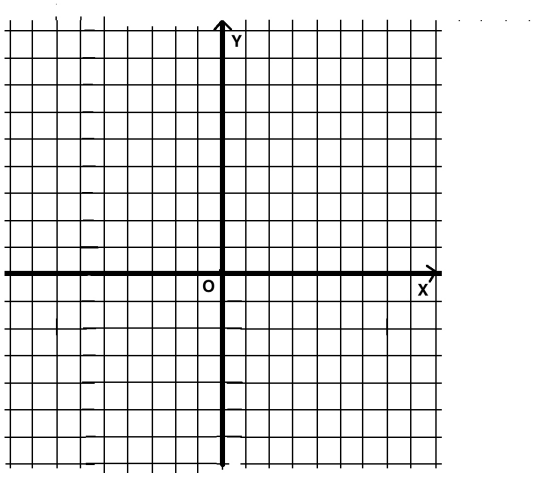
$$\begin{aligned} A &= (\quad , \quad), \\ B &= (\quad , \quad), \\ C &= (\quad , \quad), \\ D &= (\quad , \quad), . \end{aligned}$$

6. The houses of Eddie, Fred, George and Helen are also vertices a square:

   - The center of this square is at the point $O = (0, 0)$;
   - The distance from O to any of these houses is 2;
   - George's house is east of Eddie's house;
   - Fred's house is north of Helen's house;
   - Denote the vertices of this square by E, F, G, H.

   Mark the vertices on the same picture and find their addresses:

$$\begin{aligned} E &= (\quad , \quad), \\ F &= (\quad , \quad), \\ G &= (\quad , \quad), \\ H &= (\quad , \quad), . \end{aligned}$$



7. On the First Vertical street, all the houses have addresses of the form $(1, n)$, where n is a whole number.

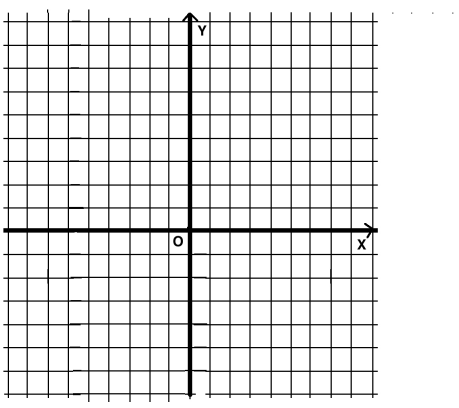
   Where is the First Vertical street?

   What can you say about the x-coordinate of all the points on the First Vertical street?

8. On the Fifth Horizontal avenue, all the houses have adddresses pf the form $(n, 5)$, where n is a whole number.

   Where is the Fifth Horizontal avenue?

   What can you say about the y-coordinate of all the points on the Fifth Horizontal avenue?



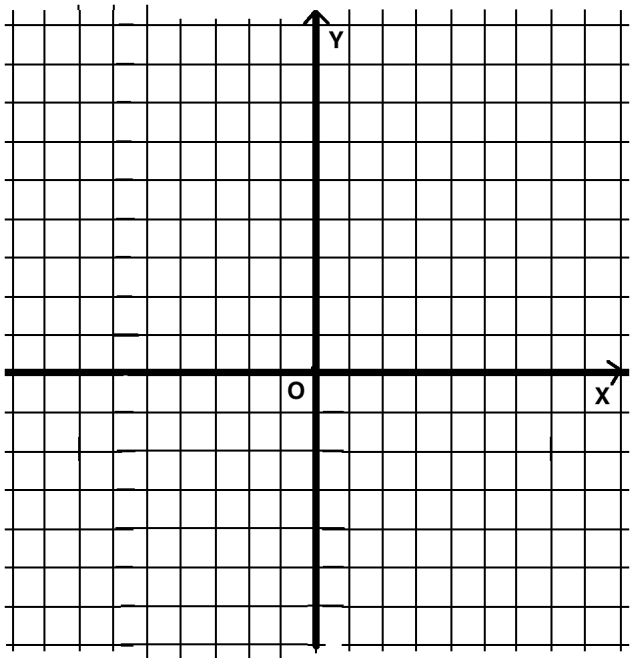
9. Several friends put together the information about their addresses into a table:

	Paul	Quinn	Ron	Sam	Tom
x	1	2	3	6	-3
y	1	2	3	6	-3

What relation is true between x and y for all the points representing their houses?

Can you write it down as a formula?

Plot the points P, Q, R, S, T representing the houses. What do you notice about their locations?



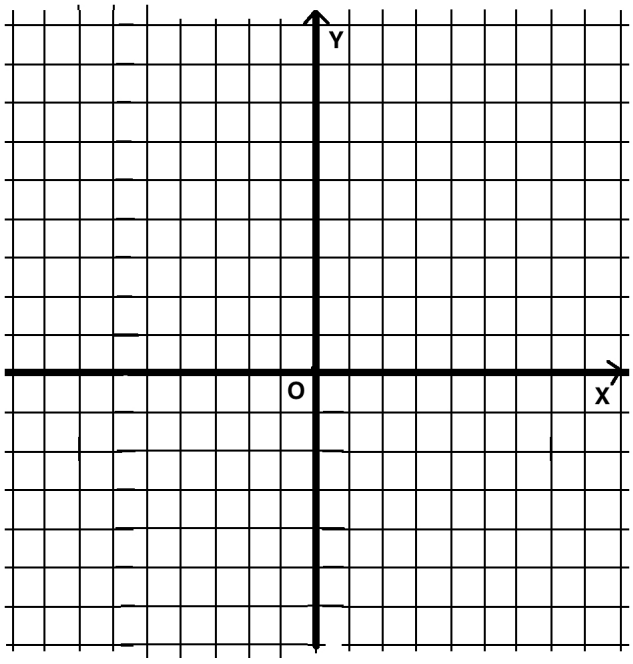
10. Several friends put together the information about their addresses into a table:

	Alex	Bob	Carla	Don	Eve
x	2	3	1	0	-1
y	4	6	2	0	-2

What relation is true between x and y for all the points representing their houses?

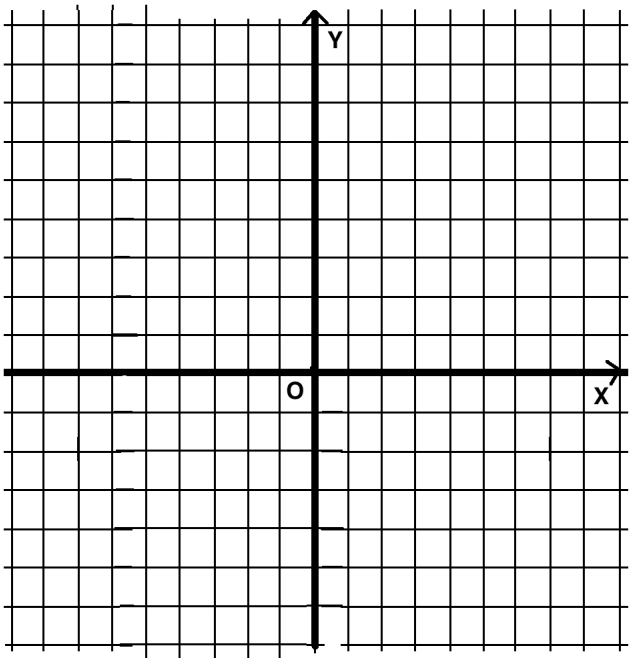
Can you write it down as a formula?

Plot the points A, B, C, D, E representing the houses. What do you notice about their locations?



11. Draw the straight line that goes through the center of the city (point $O = (0, 0)$) and the house with the address $(3, 1)$. List at least three other houses that are on this line:

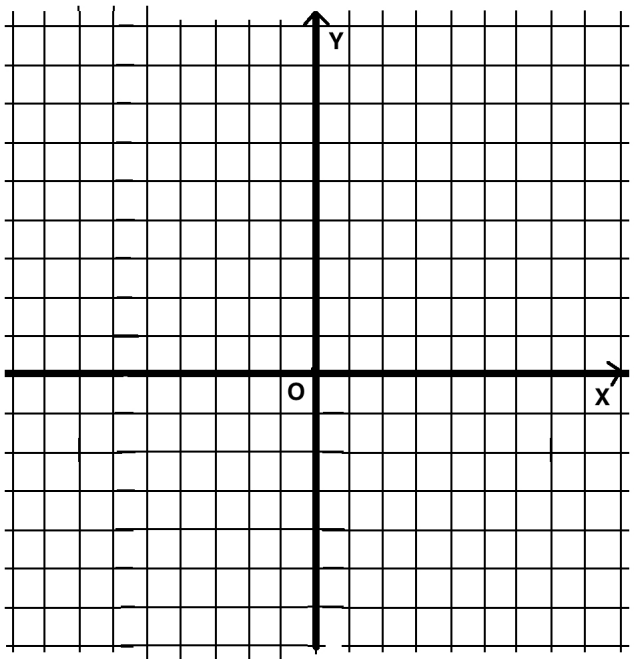
- (,)
- (,)
- (,)



What is the relation between x and y for all of these houses?

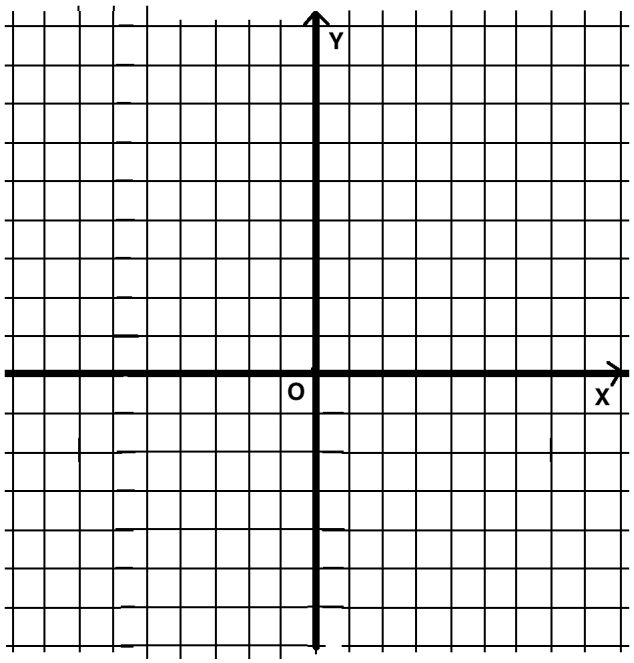
12. Find 5 houses for which $x = 2 \cdot y$. Mark them on the picture and find their coordinates.

x					
y					



13. Find 3 houses for which $y = x + 1$. Mark them on the picture and find their coordinates.

x					
y					



Do these houses lie on a line?