Exploring the City of Descartes-Part II

Junior Circle 04/03/32011

Recall that all the houses in the City of Descartes have addresses consisting of two numbers (e.g., (3, -5)):

- The first number tells you the number of the “column” the house is in. (Column numbers are positive to the right of the y-axis and negative to the left of the y-axis);
- The second number tells you the number of the “row” the house is in. (Row numbers are positive above the x-axis and negative below the x-axis);

1. For a point \((x, y)\) on the plane, the first number is called \(x\)-coordinate and the second one is called the \(y\)-coordinate. For example, for point \(P = (-1, 4)\) we have \(x = -1; y = 4\).

(a) Find the \(x\)-coordinate of the following points:
   i. \(A = (3, 2)\) \(\implies\) \(x = \) ;
   ii. \(B = (4, -5)\) \(\implies\) \(x = \) ;
   iii. \(C = (-7, -9)\) \(\implies\) \(x = \) ;

(b) Find the \(y\)-coordinate of the following points:
   i. \(A = (3, 2)\) \(\implies\) \(y = \) ;
   ii. \(B = (4, -5)\) \(\implies\) \(y = \) ;
   iii. \(C = (-7, -9)\) \(\implies\) \(y = \) ;
2. Plot several points and reflect them across the $x$-axis. (Imagine that $x$-
axis is a mirror)

(a) Does the $x$-coordinate change when you reflect a point across the $x$-
axis? If so, how?

(b) Does the $y$-coordinate change when you reflect a point across the $x$-
axis? If so, how?
3. Plot several points and reflect them across the $y$-axis. (Imagine that $y$-axis is a mirror)

(a) Does the x-coordinate change when you reflect it across the $y$-axis? If so, how?

(b) Does the y-coordinate change when you reflect it across the $y$-axis? If so, how?
4. Draw the line $y = x$.

- Reflect the following points across this line and find the coordinates of the reflections:
  
  (a) $(3, 0) \implies ( \ , \ )$
  
  (b) $(-1, 0) \implies ( \ , \ )$
  
  (c) $(0, 2) \implies ( \ , \ )$
  
  (d) $(0, -3) \implies ( \ , \ )$
  
  (e) $(1, 2) \implies ( \ , \ )$
  
  (f) $(1, 1) \implies ( \ , \ )$
  
  (g) $(3, 2) \implies ( \ , \ )$
  
  (h) $(-4, -1) \implies ( \ , \ )$

- How is the address (coordinates) of the reflection related to the address of the given point? Explain.
5. The houses of Amy, Ben, Cindy and Dan are vertices of a square:

- The center of this square is at the point $O = (0,0)$;
- The length of each of the sides of this square equals to 4;
- Amy’s house is directly to the north from Dan’s house;
- Ben’s house is east from Amy’s house;

Denote the vertices of this square by $A, B, C, D$. Mark the houses (vertices) on the picture below and find their addresses:

\[
A = (0, 4), \\
B = (4, 0), \\
C = (0, -4), \\
D = (-4, 0),
\]

6. The houses of Eddie, Fred, George and Helen are also vertices a square:

- The center of this square is at the point $O = (0,0)$;
- The distance from $O$ to any of these houses is 2;
- George’s house is east of Eddie’s house;
- Fred’s house is north of Helen’s house;
- Denote the vertices of this square by $E, F, G, H$.

Mark the vertices on the same picture and find their addresses:

\[
E = (2, 0), \\
F = (0, 2), \\
G = (0, -2), \\
H = (-2, 0),
\]
7. On the First Vertical street, all the houses have addresses of the form $(1, n)$, where $n$ is a whole number.

Where is the First Vertical street?

What can you say about the $x$-coordinate of all the points on the First Vertical street?

8. On the Fifth Horizontal avenue, all the houses have addresses of the form $(n, 5)$, where $n$ is a whole number.

Where is the Fifth Horizontal avenue?

What can you say about the $y$-coordinate of all the points on the Fifth Horizontal avenue?
9. Several friends put together the information about their addresses into a table:

<table>
<thead>
<tr>
<th></th>
<th>Paul</th>
<th>Quinn</th>
<th>Ron</th>
<th>Sam</th>
<th>Tom</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x</strong></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>-3</td>
</tr>
<tr>
<td><strong>y</strong></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>-3</td>
</tr>
</tbody>
</table>

What relation is true between $x$ and $y$ for all the points representing their houses?

Can you write it down as a formula?

Plot the points $P, Q, R, S, T$ representing the houses. What do you notice about their locations?
10. Several friends put together the information about their addresses into a table:

<table>
<thead>
<tr>
<th></th>
<th>Alex</th>
<th>Bob</th>
<th>Carla</th>
<th>Don</th>
<th>Eve</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>−1</td>
</tr>
<tr>
<td>y</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>−2</td>
</tr>
</tbody>
</table>

What relation is true between $x$ and $y$ for all the points representing their houses?

Can you write it down as a formula?

Plot the points $A, B, C, D, E$ representing the houses. What do you notice about their locations?
11. Draw the straight line that goes through the center of the city (point \( O = (0, 0) \)) and the house with the address (3, 1). List at least three other houses that are on this line:

- ( , )
- ( , )
- ( , )

What is the relation between \( x \) and \( y \) for all of these houses?
12. Find 5 houses for which $x = 2 \cdot y$. Mark them on the picture and find their coordinates.

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
13. Find 3 houses for which \( y = x + 1 \). Mark them on the picture and find their coordinates.

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Do these houses lie on a line?