Sums and averages*

April 3, 2011

1. Divide the set of numbers $\{1, 2, 9, 25, 49, 64\}$ into two subsets so that the sums of the numbers in each of them are equal.

2. Divide the strip of paper shown below into 4 parts so that the sums of numbers written on each piece of paper are the same:

1	9	16	7	12	5	4	3
8	15	10	2	13	6	11	14.

3. Annie and Jenny weigh together 40 kg. Jenny and Manny weigh together 50 kg. Manny and Benny weigh together 90 kg. Benny and Danny weigh 100 kg. Danny and Annie weigh 60 kg. How much does Annie weigh?

*Adapted from "Math Circle for grades 6 and 7" by A. Spivak (in Russian)

4. Four friends noticed that

   - without the first one, they have \$90;
   - without the second one, they have \$85;
   - without the third one, they have \$80;
   - without the fourth one, they have \$75.

   How much money does each one have?

5. The average of 2 positive whole numbers is 13.

   (a) How big can the bigger of the 2 numbers be?

   The average of 10 positive whole numbers is 7.

   (a) How big can the biggest of these 10 numbers be, if some of the numbers can be repeated?
   (b) What if we know that all 10 numbers are different–then how big can the biggest one be?

6. On his first four tests, Arnold scored 60, 73, 80, and 67. On his fifth test, he got a perfect score and increased his overall test average by 2 points. What is a perfect score? (Note: A perfect score is the same on all 5 tests.)

7. The average age of the players in an 11 player soccer team is 22 years old. If you don't count the captain, the average age of the other players is 21 years old. How old is the captain?

8. Katie, Helen, Mary, and Nina were singing at a concert. Each song was performed by 3 of the girls singing together. Katie sang the most songs, 8. Nina sang the fewest songs, 5. How many songs did they perform altogether?

9. Tim's average score on 12 assignments is 3.5 out of 5.

   (a) Is it possible that Tim received 5 points on any of the assignments?

   (b) Is it possible he received 0 points on any of the assignments?

   (c) What is the fewest possible number of assignments he could redo so that his average becomes 4?

   (d) What is the most he might have to do? (Assume he gets 5 out of 5 each time he re-does an assignment.)

10. Find a formula, in terms of n, for the average of the first n positive whole numbers. Check the first several values of n to make sure your formula works.