The Triangle Inequality

If ABC is a triangle, then the following inequalities are true:

$$AB < BC + AC \qquad BC < AB + AC \qquad AC < AB + BC$$

Note that we do not consider three collinear points to form a triangle. What happens to the inequalities when the points A, B, C are collinear?

Problems

1. Prove that the length of any side of a triangle is less than half the perimeter of the triangle.
2. Prove that for any three points A, B, C

$$AB \geq |AC - BC|.$$

3. Side AC of triangle ABC has length 3.3, and side BC has length 0.6. If the length of side AB is an integer, what is that length?
4. It is 6850 miles from Los Angeles to Nanchang, China; 300 miles from Los Angeles to Santa Cruz, CA; 5700 miles from Santa Cruz to Seoul, South Korea; and 850 miles from Seoul to Nanchang, China. How far is it from Los Angeles to Seoul?
5. Suppose $ABCD$ is a convex quadrilateral. What is the point inside $ABCD$ such that the sum of the distances from the point to the vertices of the quadrilateral is as small as possible?
6. A billiard ball lies on a table in the shape of an acute angle. How should you hit the ball so that it returns to its starting location? Is it always possible to do so?
7. A point A lies inside an acute angle. If A is reflected through the sides of the angle to obtain points B, C, and D, E are the points where line BC intersects the sides of the angle, prove that $BC/2 > DE$.

8. A point C lies inside a right angle with vertex O. If A, B are points on the two sides of the right angle, prove that the perimeter of triangle ABC is at least twice the distance OC.

9. If O is a point inside triangle ABC, prove that $AO + OC < AB + BC$.

10. You have parked at a major department store, and are heading to the entrance, which lies several lanes of cars down from where you parked. When squeezing between cars you can not walk diagonally, but in between the cars you can walk diagonally. What is the fastest way to reach your destination?

11. Two villages lie on opposite sides of a river whose banks are straight lines. A bridge is to be built over the river perpendicular to the banks. Where should the bridge be built so that the path from one village to the other is as short as possible?