

How To Divide Up Seats Among Pirates, and More

1. Liz and Daniel want to split a cake between them (they are very hungry). But they want to make sure that each person gets at least half the cake, *from that person's own perspective*. So Liz cuts the cake into two pieces of equal size (from her perspective), and Daniel chooses the one which is bigger (from his perspective). So each of them gets at least half the cake, based on their own judgment!

How can *three* people do a similar “cut-and-choose” procedure, so that each person gets at least $\frac{1}{3}$ of the cake, from their own perspective?

2. Each pirate ship is going to send representatives to serve in the pirates' governing body, the Assembly of Rovers, Raiders, and Rogues (ARRR). There are **10 seats** in the Assembly, and each of the 4 ships should get a number of seats proportional to its crew size. Here are the ships and crew sizes:

Ship	Crew Size
Queen Anne's Revenge	60
Jolly Roger	45
Black Pearl	30
Good Ship Lollipop	15

How many seats should each ship get?

3. A few years later, the pirate ships' crew sizes have changed, and they agree to recalculate how many seats each ship will receive.

Ship	Crew Size
Queen Anne's Revenge	77
Jolly Roger	46
Black Pearl	30
Good Ship Lollipop	27

Now how would you divide up the 10 seats among the ships, *and why*?

Several of the ships have offered competing proposals for apportioning seats. All of the proposals calculate a *divisor* $D = (\text{total \# of pirates})/(\text{\# of seats})$. (So in this case $D = 180/10 = 18$.) Then, for each ship, that ship's *quota* Q of seats is computed as $Q = \text{crew size}/D$. (So for instance Queen Anne's Revenge has $Q = 77/18 \approx 4.28$.)

Proposal 1 (Captain Jack Sparrow, Black Pearl):

1. For each ship, **round its quota Q down** and assign the ship that many seats to start with. Call the leftover fractional part of Q LEFTOVER. (For instance, if $Q = 4.6$, that ship gets 4 seats to start and that ship's value of LEFTOVER is .6)
2. If there are still seats left to be assigned, give them to the ships with the **largest LEFTOVER value(s)**.

Proposal 2 (Blackbeard, Queen Anne's Revenge):

1. For each ship, **round its quota Q down** and assign the ship that many seats to start with.
2. If there are still seats to be assigned, **decrease D** as necessary and repeat step 1 until D is small enough that exactly the right number of seats are assigned.

Proposal 3 (Shirley Temple, Good Ship Lollipop):

1. For each ship, **round its quota Q up** and assign the ship that many seats to start with.
2. If more than S seats have been assigned, **increase D** as necessary and repeat step 1 until D is large enough that exactly the right number of seats are assigned.

Proposal 4 (Captain Hook, Jolly Roger):

1. For each ship, **round its quota Q to the nearest whole number** and assign the ship that many seats to start with.
2. If there are still seats remaining, **decrease D** as necessary and repeat step 1 until D is small enough that exactly S seats are assigned. If on the other hand, more than S seats have been assigned, **increase D** as necessary and repeat step 1 until D is large enough that exactly the right number of seats are assigned.

For each ship, calculate its quota and then figure out how many seats it would get under each plan. You can use the following table to record your answers:

Ship	Crew Size	Quota	Seats under...			
			Proposal 1	Proposal 2	Proposal 3	Proposal 4
Queen Anne's Revenge	77	4.28				
Jolly Roger	46					
Black Pearl	30					
Good Ship Lollipop	27					

In the U.S. House of Representatives, each state gets a number of representatives proportional to its population. For instance, California is the largest state with about 37,000,000 people, so we get the most seats (53). Wyoming, the smallest state with only 550,000 people, gets the fewest representatives (only 1).

4. (a) As mentioned above, Wyoming has only 1 seat (and therefore 1 congressional district) for its 550,000 people. Montana also has only 1 seat (and 1 district) with a population of about 970,000. What is the average number of people for each district in California? (Use 2 digits of accuracy, such as “about 37,000,000” rather than “37,249,105.”)

 - (b) The current U.S. population is about 310,000,000. There are 435 members of the House of Representatives. On average, how many people are in each Congressional district in the U.S.? (Use 2 digits of accuracy.)

 - (c) Residents of Washington, D.C. don't get a representative (since D.C. is not a state). Guess how many people live there?

 - (d) According to the first Census, the United States in 1790 had about 4,000,000 people (and only 13 states!). At this time, the number of seats in the House was 65. On average, how large was each district (2 digits of accuracy)?
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5. The number of Representatives per state is recalculated every 10 years. Based on the 1990 census, Oklahoma had 6 House seats. According to the 2000 census, Oklahoma's population grew by almost 10 percent between 1990 and 2000 (from 3.15 million to 3.45 million). But Oklahoma's number of seats went down, from 6 to 5. What is the most likely explanation?

6. Every weekend, Cinderella and her stepsisters go to balls. Some weekends there are two balls scheduled at the same time, and the stepsisters often argue about which one to go to. Cinderella’s stepmother has finally gotten so sick of it, she tells them whenever any of the sisters can’t decide which ball to go to, the 3 of them must vote on what to do. But she adds, “Cinderella, since you are not as smart (or as pretty!) as Lucinda and Florinda, your vote only counts for 1, and their votes count for 5 each.” Should Cinderella complain?
7. After some time, the pirate ships’ crews have changed, and there are two new ships. They decide to try a new system for ARRR: Each ship will have only one actual representative, but that person will get a number of *votes* proportional to their ship’s crew size, where a majority of votes is necessary to carry any motion.

Here are the ships, along with the number of votes the pirate who represents that ship will get:

District	Number of Votes
Queen Anne’s Revenge	9
Jolly Roger	9
Black Pearl	7
The Good Ship Lollipop	3
Tarantula	1
Fancy	1

It so happens that these numbers are perfectly proportional to the respective crew sizes. Nevertheless, some upstarts on The Good Ship Lollipop, the Tarantula, and the Fancy feel that the system is unfair, and they’ve hired you as their lawyer to sue the other ships. Make an argument that the system is unfair to these ships (even though they have a proportional number of votes!).