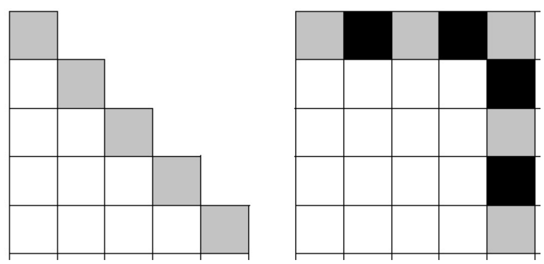
Triangular and Square Numbers

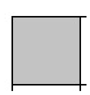
October 17, 2010



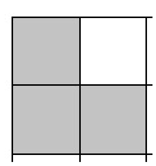
1. Triangular Numbers

1. The block triangles we will be looking at have the special property that both the height and the width are equal. Numbers that represent the total number of boxes that make up these special triangles are called *triangular numbers*.

    (a) The first triangular number, which we denote by T_1, is 1:

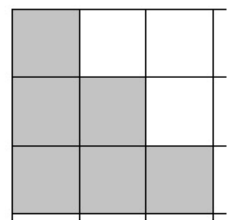
    

    (b) The second triangular number, which we denote by T_2, is 3:

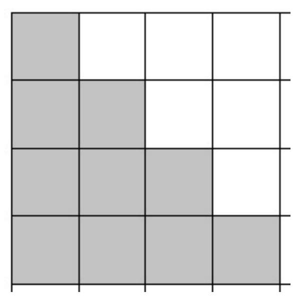
    i. Why can 2 not be a triangular number?

(c) The third triangular number, which we write as T_3, is 6.

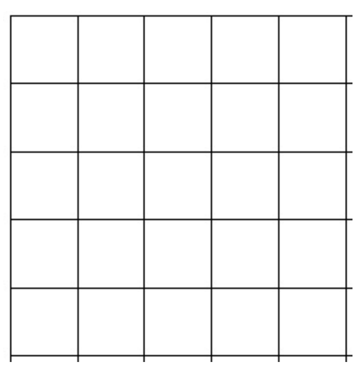


i. Why are 4 and 5 not triangular numbers?

(d) What is T_4, the fourth triangular number?

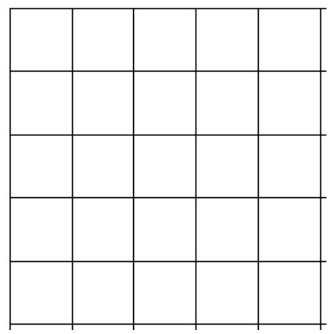


(e) What is T_5, the fifth triangular number? (You may find it helpful to draw the next-largest block triangle in the triangle below.)

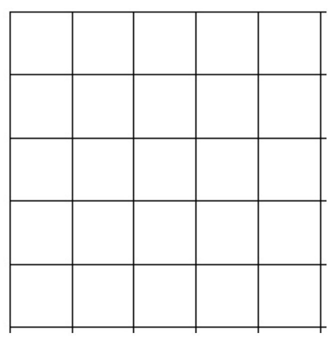


2. Remember from last week our definitions of *similar* shapes: two shapes are similar when the larger is a "magnified" version of the smaller shape.

   (a) Draw the triangle corresponding to T_3 below:

   (b) Draw the triangle corresponds to T_4 below:

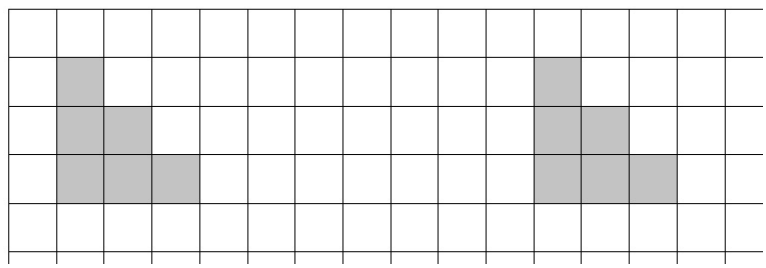
   

   (c) Compare the two shapes.

      i. What do they have in common?

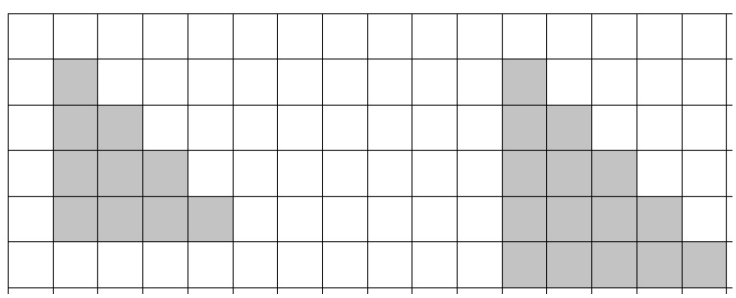
      ii. Do you think that they are *similar* by our definition from last week? (Is the larger triangle the same as the smaller triangle, magnified twice or three times? How many stairs does each have?)

(d) Even though the T_3-triangle and the T_4-triangle are not *similar* by our definition last week, we can speak of a *gnomon* that transforms T_3-triangle into T_4-triangle. Draw two different gnomons on each of the T_3-triangles below that make it into a T_4-triangle. (Position one of the gnomons "below" the T_3-triangle and the other along the "stairs" of the T_3-triangle)



i. How many boxes did the gnomons add to turn the T_3-triangles into T_4-triangles? (What is the size of the gnomon?)

3. For now, consider only the gnomons positioned along the "stairs". Below are two more block triangles.



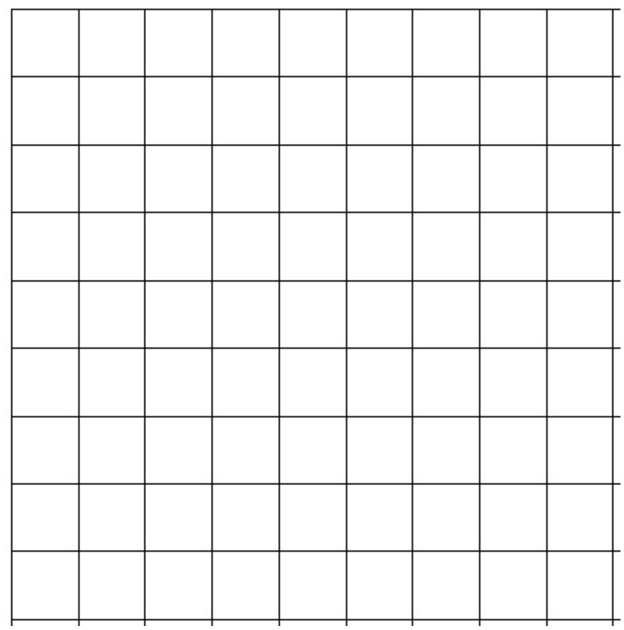
(a) What triangular numbers do the block triangles correspond to? Label the triangles.

(b) Draw in the gnomon that will make the smaller block triangle into the larger.

i. How many blocks did you add this time? (What is the size of the gnomon?)

4. Can you guess the size of the gnomon that would be needed to turn the T_5-triangle into the T_6-triangle?

(a) Draw it out and verify your answer



(b) What do you think about the pattern of the sizes of gnomons when we change a T_n-triangle into the T_{n+1}-triangle. That is, how is the triangular number associated with a certain block triangle relate to the size of the gnomon needed to make it into the next-largest triangle?

5. Write down the first 6 triangular numbers:

(a) T_1=

(b) T_2=

(c) T_3=

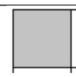
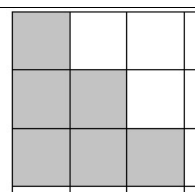
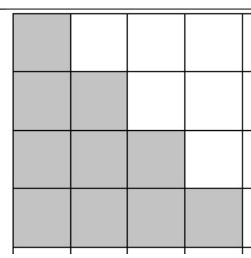
(d) T_4=

(e) T_5=

(f) T_6=

6. Is there a pattern to finding a certain triangular number?

(a) Write out a way that you can add up the number of boxes in each row or column to find the triangular number that is associated with that triangle:

	Triangle	Number	Addition
T_1			
T_2			
T_3			
T_4			

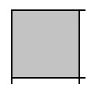
(b) Can you say what the 9th triangular number is without drawing a triangle?

7. Draw a triangle corresponding to T_9 and verify your answer above.

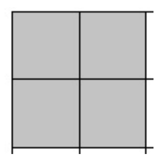
2. Square Numbers

There is another special set of numbers known as *square numbers.* As you might guess from their name, these numbers represent the number of blocks contained inside of a square.

1. Just like in triangular numbers, the first square number, which we note as S_1, is 1.

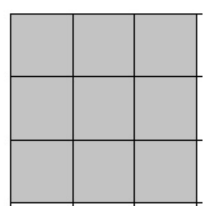


   (a) The second square number, which we note as S_2, is 4.



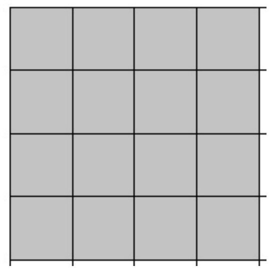
      i. Why are 2 and 3 not square numbers?

   (b) The third square number, which we write as S_3, is 9.

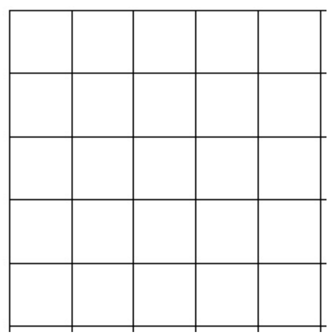


      i. Why are 5, 6, 7 and 8 not square numbers?

(c) What is S_4, the fourth square number?



(d) What is S_5, the fifth square number? (You may find it helpful to draw the next-largest block triangle in the triangle below.)

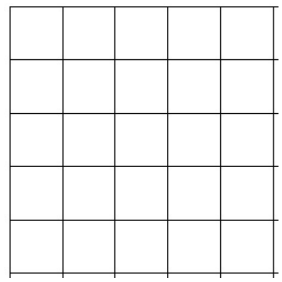


2. Again, remember from last week our definitions of *similar* shapes: two shapes are similar when the larger is a "magnified" version of the smaller shape.

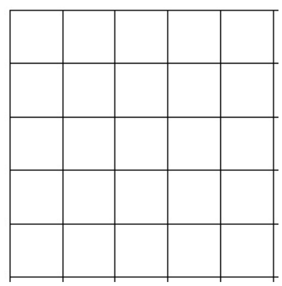
   (a) Do you think all squares are *similar* each other by this definition? Why or why not?

3. Consider the the squares which correspond to S_3 and S_4.

   (a) Draw the S_3−square below.



   (b) Draw the S_4−square below

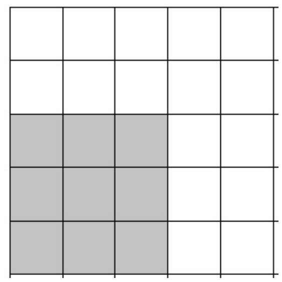


   (c) Compare the two shapes.

      i. What do they have in common?

      ii. Since they are similar, is there a piece we can add to the smaller square to make it look like the larger square? (Do you think there is a gnomon?)

(d) Draw a gnomon on each the S_3-square below that makes it into an S_4-square.

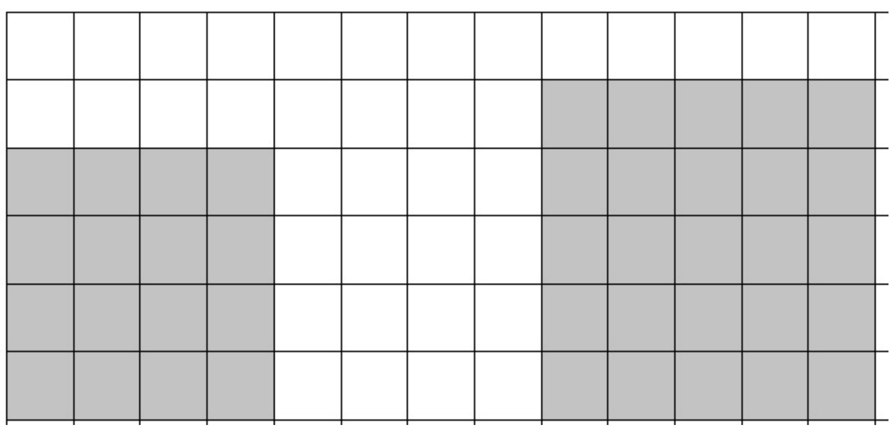


i. Was the gnomon connected or disconnected?

ii. What is the gnomon shaped like?

iii. How many boxes did the gnomons add to turn the S_3-square into S_4-square? (What is the size of the gnomon?)

4. Below are two more squares which are also *similar*.



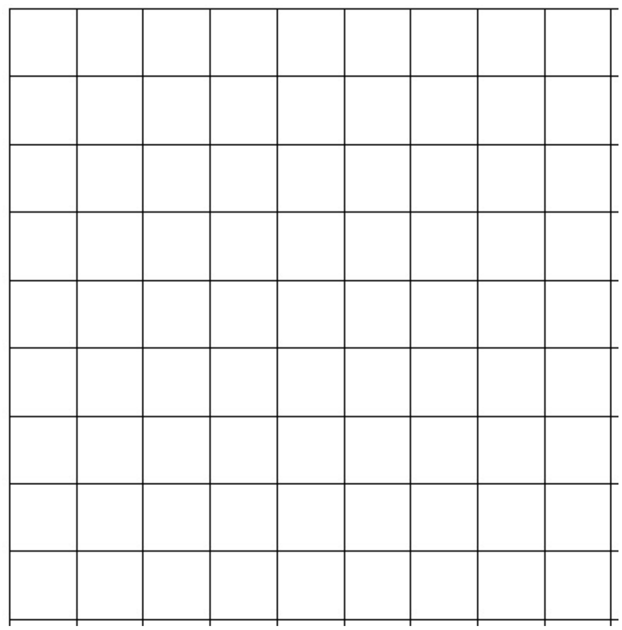
(a) What square numbers do the squares above correspond to? Label the squares.

(b) Draw in the gnomon that makes the smaller square into the larger.

i. How many blocks did you add this time? (What is the size of the gnomon?)

5. Can you guess the size of the gnomon that would be needed to turn the S_5-square into the S_6-square?

   (a) Draw it out and verify your answer

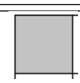
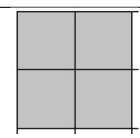
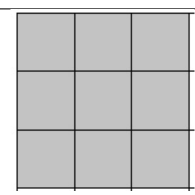
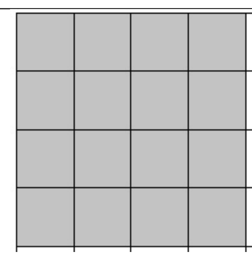
   

   (b) What do you think about the pattern of the size of gnomons when we change a S_n-square into the S_{n+1}-square. That is, how is the square number associated with a certain square relate to the size of the gnomon needed to make it into the next-largest square?

6. Write down the first five square numbers:

   (a) S_1=
   (b) S_2=
   (c) S_3=
   (d) S_4=
   (e) S_5=

7. Is there a pattern to finding a certain square number? Write out a way that you can add up the number of boxes in each row or column to find the square number that is associated with the same size as the square. Next, write a way you can multiply the number of boxes in each row and column to get the same answer:

	Square	Number	Addition	Multiplication
S_1				
S_2				
S_3				
S_4				

(a) Can you say what the 9th square number is without drawing the square?

3. How are triangular and square numbers related?

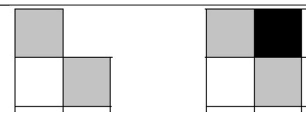
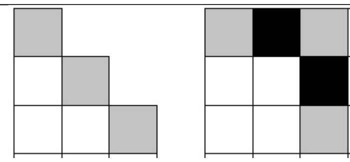
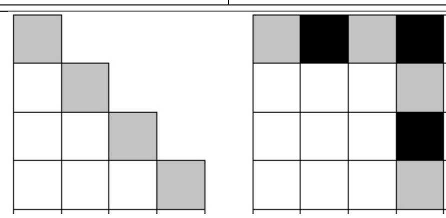
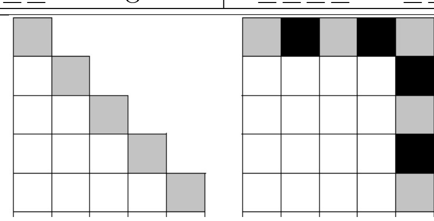
1. Draw out the first three triangular numbers next to the first three square numbers. Do you notice any patterns?

   (a) T_1 and S_1

   (b) T_2 and S_2

   (c) T_3 and S_3

2. Consider the gnomons added to T_n and S_n to make the next-largest block triangle or square. Assume the white boxes make up the original squares, the gray and black boxes make up the gnomons.

	
___ to ___ triangles	____ to ____ squares
	
___ to ___ triangles	____ to ____ squares
	
___ to ___ triangles	____ to ____ squares
	
___ to ___ triangles	____ to ____ squares

For all of these pairs, how are the sizes of the gnomons for the block triangles related to the sizes of the gnomons for the squares? In particular, how are the light-gray boxes in the box triangle's gnomons related to the light-gray squares present in the square's gnomons?

Homework

Find a real-life example of when triangular numbers or square numbers occur. Come next week prepared to share with your table.