

Topics in Plane Geometry

May 31, 2015

Perpendicular Lines

Right angle = 90°

Acute angle $< 90^\circ$

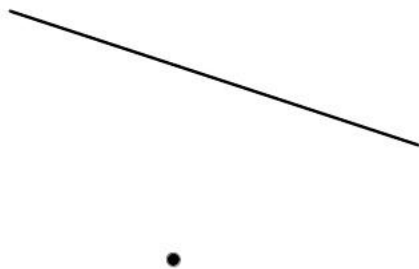
Obtuse angle $> 90^\circ$

1. Identify the angles below and circle the correct label:

2. Draw a line perpendicular to the given line, going through the given point.



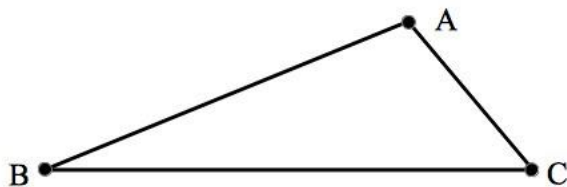
3. Draw a line perpendicular to the given line, going through the given point.



Area of a triangle

1. Area of a Triangle via Area of a Rectangle:

- (a) Label the vertices of the triangle below so that the top vertex is A .
- (b) Draw an altitude AH from the vertex A to the opposite side, BC . Also, label the point H .



$\triangle ABC$ is now divided into two right angle triangles, $\triangle AHB$ and $\triangle AHC$.

- (c) Shade in $\triangle AHB$.
- (d) On the picture above, draw the line through A which is parallel to BC . Draw the lines through B and C which are perpendicular to this line. Denote points of intersection by D and E .
- (e) What shape is the quadrilateral $BCED$?
- (f) $BCED$ is broken into 4 triangles:
 - . Shade in $\triangle AHC$.

(g) What fraction of the area of the rectangle is taken by $\triangle ABC$? Why?

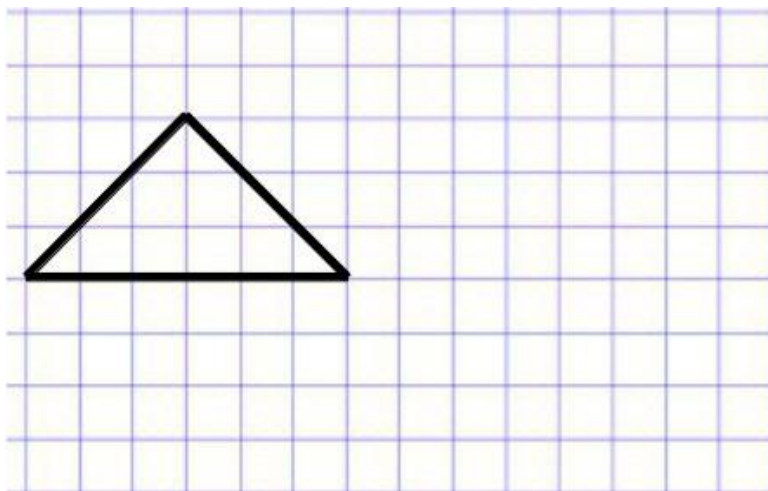
(h) Write down an expression for the area of the rectangle if $AH = h$ and $BC = b$:

$$\text{Area}(\text{rectangle}) =$$

(i) Write down an expression for the area of $\triangle ABC$:

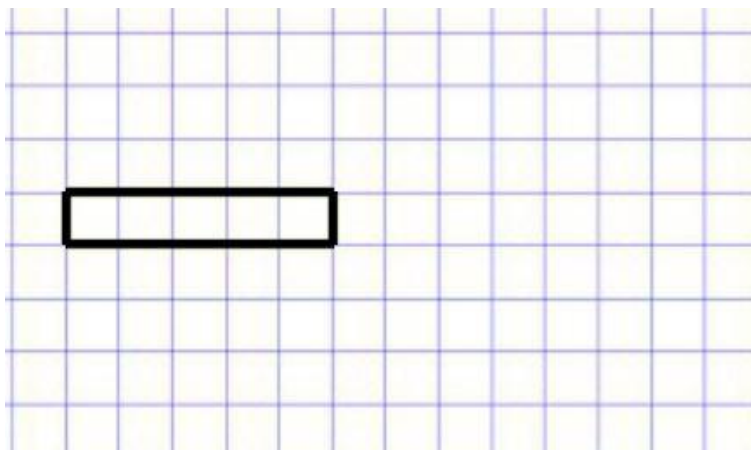
$$\text{Area}(\triangle ABC) =$$

2. Draw a square which has the same area as the triangle below.

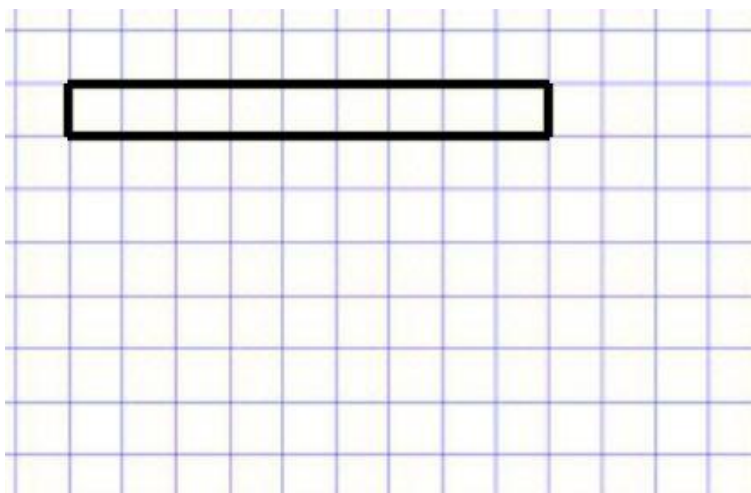


(a) Explain why the areas are the same:

3. Draw a square which has the same perimeter as the rectangle below:



- (a) Explain how you made the perimeters the same:
- (b) Which shape has a bigger area: the square or the rectangle?
4. Draw a square which has the same area as the rectangle below:



- (a) Which shape has a bigger perimeter: the square or the rectangle?

Isoperimetric problem

Suppose you are given 40 meters of fencing and you want to surround a rectangular plot of land whose area is as large as possible. What should the sides of the rectangle be?

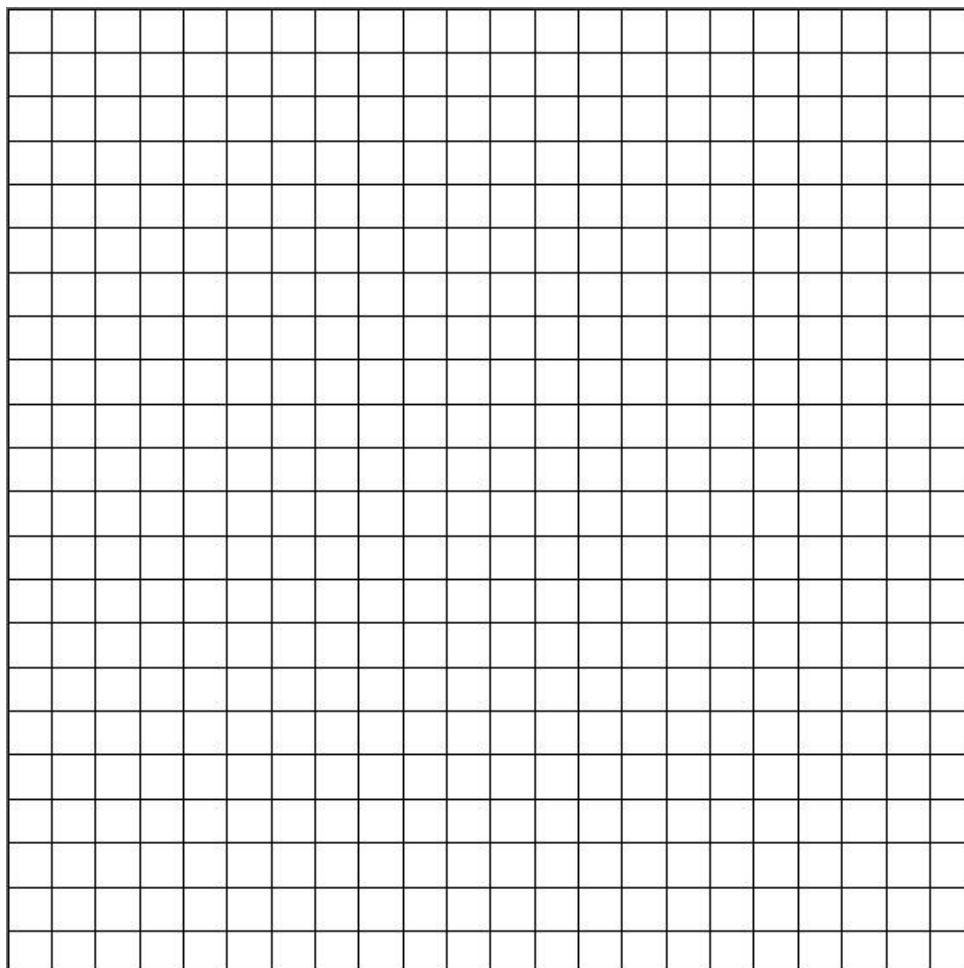
Let's explore how large of a plot we can surround with 40 meters of fencing.

1. Start by filling out the table below:

base	height	area
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		

19		
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2. What is the shape of the rectangle with the biggest area?
3. Do you think this will hold in general?
4. Use the grid paper below to draw a right triangle with a base of 8 and a height of 6. Next, draw an equilateral triangle with side lengths of 8. Draw the second triangle so that the bases of the two triangles coincide.



(a) The perimeters of these two triangles are the same. Which do you think has the larger area?

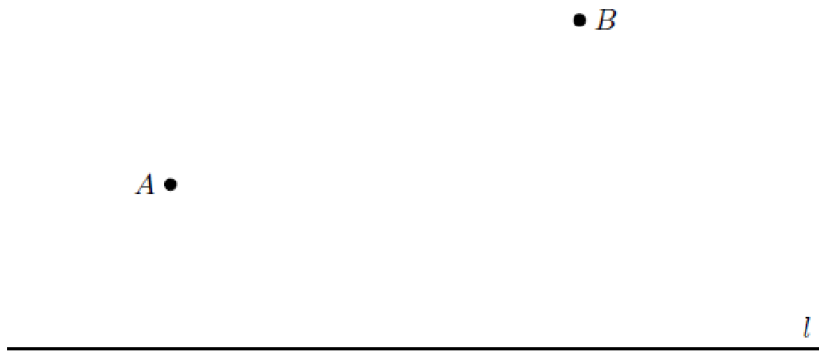
Practice Problems

1. Using a compass and a ruler, construct the shortest possible path from the point A to the straight line l below.

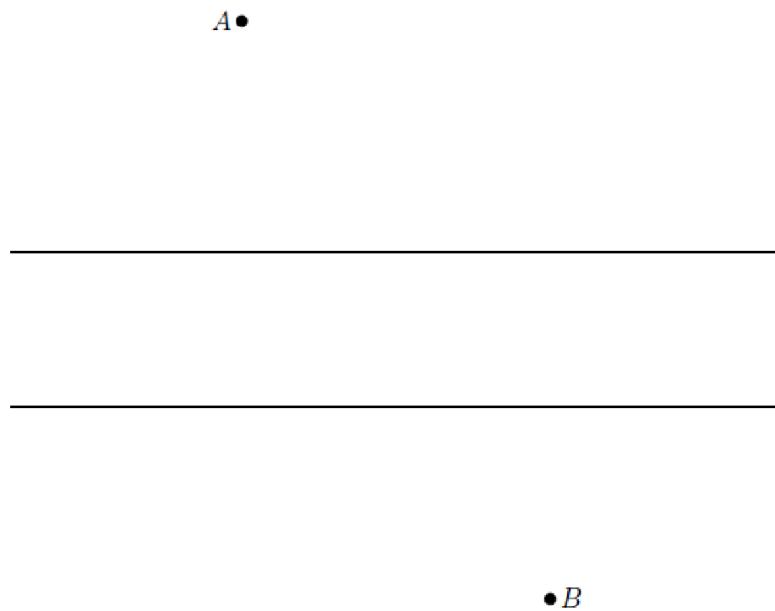
• A



2. The cities A and B are located on one side of a straight highway. Use a ruler to construct the shortest possible road connecting A to the highway and then to B.



3. The cities A and B are separated by a river having parallel straight banks. Using a ruler, construct the shortest possible highway connecting A to B . The bridge across the river must be perpendicular to the banks.



Warmup: Magic Numbers

Choose a 3-digit number:

Write it down backwards:

Subtract the smaller number from the larger. If you end up with a 2-digit number, put a 0 in front (ie. 42 is written as 042):

Write the number you calculated above backwards:

Now add the previous two numbers together:

What did you get? Compare with a neighbor and try it again with a different 3-digit number. What do you observe?