# Lesson 6: Young Tableaux

Konstantin Miagkov

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**Definition 1.**
The *partition number* $p(n)$ for a positive integer $n$ is the number of partitions of $n$ into positive parts where partitions different only in ordering of the summands are not distinguished.

**Problem 1.**
Compute $p(n)$ for all $n$ from 1 to 10.

**Problem 2.**
You need to pack cookies into boxes. There are 10 boxes each of which can contain at most 3 cookies. How many ways are there to put 22 cookies into boxes (leaving no box empty)? The boxes are indistinguishable.

**Problem 3.**
Show that the number of partitions of $n$ into at most $k$ parts each of which is at most $\ell$ is equal to the number of partitions of $n$ into at most $\ell$ parts each of which is at most $k$.

**Problem 4.**
Show that the number of partitions of $n$ into $k$ parts is equal to the number of partitions of $n + \binom{k}{2}$ into $k$ *distinct* parts.

**Problem 5.**
Show that the number of partitions of $n$ into distinct odd parts is equal to the number of partitions of $n$ such that their Young tableaux are symmetric with respect to the diagonal.

**Problem 6.**
Let the side lengths of the triangle $\triangle ABC$ be $a, b, c$ where $a$ is the length of $BC$, $b$ is the length of $AC$ and $c$ is the length of $AB$. Let $M, N$ be points on $AB$ and $BC$ respectively such that $AM = BN$ and $MN \parallel AC$. Find the length of $MN$ in terms of $a, b, c$.

**Problem 7.**
Consider points $A, B, C, D$ on a line $\ell$ in that order. Draw two parallel lines through points $A$ and $B$, and another pair of parallel lines through points $C$ and $D$. The two pairs of parallel lines create a parallelogram. Consider the two points at which the lines containing the diagonals of this parallelogram intersect $\ell$. Show that these two points do not depend on the choice of the two pairs of parallel lines.