

# Homework 5: Combinations and Pascal's Triangle

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## 1 Homework

### Problem 1.

How many ways are there to distribute seven white and two black balls into 9 holes? The balls of the same color are indistinguishable, the holes are distinguishable, and some holes can remain empty.

### Problem 2.

In a convex quadrilateral  $ABCD$  the points  $E, F, G$  are midpoints of  $AB, BC$  and  $AD$  respectively. Given that  $GE \perp AB$  and  $GF \perp BC$ , find the  $\angle ACD$ .

## 2 Reading

### Solution 1 (L4.5).

Let  $AA_1$  and  $BB_1$  be altitudes in a triangle  $\triangle ABC$ . Show that  $CA_1 \cdot CB = CB_1 \cdot CA$ .

*Proof.* Note that  $\angle AA_1B = \angle AB_1B = 90^\circ$ , therefore  $AB_1A_1B$  is a cyclic quadrilateral. Then  $CA_1 \cdot CB = CB_1 \cdot CA$  as the power of a point  $C$  with respect to the circumcircle of  $AB_1A_1B$ .  $\square$

### Solution 2 (H5.1).

How many ways are there to split 200 passengers into 3 train carts?

*Proof.* From the lesson we know that the number of ways to split them so that no cart is empty is  $\binom{199}{2} = 199 \cdot 198/2 = 19701$ . If we leave two carts empty, we have 3 more ways. And finally, if we leave one cart empty, we have 199 choices to fill the other two carts, and three choices to pick the empty cart. Thus the final answer is  $19701 + 3 \cdot 199 + 3 = 20301$ .  $\square$