Homework 3: Combinations and Pascal's Triangle

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1 Homework

Problem 1.
How many ways are there to arrange 2 green, 2 red and 2 blue balls in a row so that not two balls of the same color are adjacent to each other?

Problem 2.
Let T be a point inside a circle, and A, B, C, D be points on the circle such that lines AB and CD intersect at T. Show that $AT \cdot TB = CT \cdot TD$.

2 Reading

Solution 1 (L2.3)**.**
If the ratios of three binomial coefficients are

$$\binom{n+1}{m+1} : \binom{n+1}{m} : \binom{n+1}{m-1} = 5 : 5 : 3$$

find n, m. Since

$$\binom{n+1}{m+1} : \binom{n+1}{m} : \binom{n+1}{m-1} = 5 : 5 : 3$$

we know that

$$\binom{n+1}{m+1} = \binom{n+1}{m}$$

Writing it out we get

$$\frac{(n+1)!}{(m+1)!\,(n-m)!} = \frac{(n+1)!}{(m)!\,(n+1-m)!}$$

$$(m)!\,(n+1-m)! = (m+1)!\,(n-m)!$$

Cancelling out $m!\,(n-m)!$ we get $n+1-m = m+1$ or $n = 2m$. Now we use

$$\binom{n+1}{m} : \binom{n+1}{m-1} = 5 : 3$$

to do the same factorial calculations:

$$\frac{3(n+1)!}{m!\,(n+1-m)!} = \frac{5(n+1)!}{(m-1)!\,(n+2-m)!}$$

$$3(m-1)!\,(n+2-m)! = 5m!\,(n+1-m)!$$

Cancelling out $(m-1)!\,(n+1-m)!$ we get $3(n+2-m) = 5m$. Plugging in $n = 2m$ we have

$$3m + 6 = 5m$$

$$m = 3, n = 6$$

and we are done.

Solution 2 (H2.1)**.**
If we want to descend to the n-th row of Pascal's triangle, we have to make n total steps, since every step takes us one row down. At each step we can choose to go left or right, so the number of ways is 2^n.