13 Decimal and binary numbers

Materials for the lesson and homework: a couple of regular pencils, an eraser, a pencil sharpener; an abacus.

Warm-up

Problem 13.1 *Alice has a drawer with ten pairs of white socks and twelve pairs of blue socks. How many socks does the girls need to take out of the drawer without looking to make sure she has taken a pair of white socks? Hint: consider the worst case possible.*

Problem 13.2 *Alice has a drawer with ten pairs of white socks and twelve pairs of blue socks. How many socks does the girls need to take out of the drawer without looking to make sure she has a pair of socks of matching color?*

Lesson

In this lesson we will examine similarities between decimal and binary numbers.

Question 13.1 *How many digits are there in the decimal system? What are the digits?*

Question 13.2 *How many digits are there in the binary system? What are the digits?*

Recall that the numbers 1, 2, 4, 8, 16, 32, etc. are basic for the Binary Land way of counting.

Problem 13.3 *What are the decimal values of the following binary numbers?*

$B\ 1 =$ _____ $B\ 10 =$ _____

$B\ 100 =$ _____ $B\ 1000 =$ _____

$B\ 10000 =$ _____ $B\ 100000 =$ _____

Problem 13.4 *How do the basic numbers* 64 *and* 128 *look in the binary notations?*

$64 = B$ _____

$128 = B$ _____

Question 13.3 *What numbers would you call basic in the decimal system?*

Note that a decimal number is nothing but a code telling you how many basic decimal numbers you need to take. For example,

$$462 = \text{four 100s} + \text{six 10s} + \text{two 1s}.$$

It is convenient to write this decomposition in a table very similar to the tables of binary weights we have used before.

studied number	*basic decimal numbers*				
	10000	**1000**	**100**	**10**	**1**
462			4	6	2

To see the decomposition even better, make the number 462 on the abacus.

Problem 13.5 *Break the number* 73 *into a sum of basic decimal numbers. Write the coefficients in the table below. Then make the number on the abacus and check that it shows the same.*

studied number	*basic decimal numbers*				
	10000	**1000**	**100**	**10**	**1**
73					

Problem 13.6 *Break the number* 2019 *into a sum of basic decimal numbers. Write the coefficients in the table below. Then make the number on the abacus and check that it shows the same.*

studied number	*basic decimal numbers*				
	10000	**1000**	**100**	**10**	**1**
2019					

Problem 13.7 *Find the binary notation for the number* 73.

studied number	*basic binary numbers*						
	64	**32**	**16**	**8**	**4**	**2**	**1**
73							

$73 = B$ _____

Question 13.4 *What is in common between the decimal decomposition of the number* 73 *in problem 13.5 and the binary decomposition of the number in problem 13.7? What is different?*

Problem 13.8 *Find the binary notation for the number* 234.

studied number	*basic binary numbers*							
	128	**64**	**32**	**16**	**8**	**4**	**2**	**1**
234								

$234 = B$ _____

Problem 13.9 *Break the number* 234 *into a sum of basic decimal numbers. Write the coefficients in the table below. Then make the number on the abacus and check that it shows the same.*

studied number	*basic decimal numbers*				
	10000	**1000**	**100**	**10**	**1**
234					

Question 13.5 *What is in common and what is different between the decompositions of the number* 234 *in problems 13.8 and 13.9?*

Let us summarize the comparison of decimal and binary numbers below.

Basic decimal numbers	**Basic binary numbers**
• 1	• B 1 = 1
• 10 equals ten 1s.	• B 10 = 2 equals two B 1s.
• 100 equals ten 10s.	• B 100 = 4 equals two B 10s.
• 1000 equals ten 100s.	• B 1000 = 8 equals two B 100s.
• 10000 equals ten 1000s.	• B 10000 = 16 equals two B 1000s.

In the decimal system, there are ten digits. Except for 1, a basic decimal number equals ten of the previous basic decimal numbers. The base of the system is ten.

In the binary system, there are two digits. Except for 1, a basic binary number equals two of the previous basic binary numbers. The base of the system is two.

Question 13.6 *Do people use other bases?*

Yes, they do. The most important numbers for computer science are binary. The next in importance are octal (base 8) and hexadecimal (base 16). We will study them later.

Problem 13.10 *Explain the following sentence.*

There are 10 types of people: those who know binaries and those who don't.

Now is the time to learn counting in the binary system.

0, 1, and ... ups, we are out of single digit numbers because there are only two digits in the binary system, 0 and 1.

Problem 13.11 *What is the next binary number?*

The next binary number is _____ .

0, 1, 10, 11, and ... we are out of two-digit numbers.

Problem 13.12 *What is the next binary number?*

The next binary number is _____ .

0, 1, 10, 11, 100, 101, 110, 111, ... Now we are out of three-digit numbers.

Problem 13.13 *What is the next binary number?*

The next binary number is _____ .

Problem 13.14 *Continue counting in the binaries:*

0, 1, 10, 11, 100, 101, 110, 111, _____ ,

_____ , _____ , _____ , _____ , _____

Problem 13.15

- *What is the value of the bold-face digit* **5** *in the decimal number* **5**1*?*

  The value is _____ .

- *What is the value of the bold-face digit* **5** *in the decimal number* 2**5**34*?*

  The value is _____ .

- *What is the value of the bold-face digit* **5** *in the decimal number* 20**5***?*

  The value is _____ .

- *Does the value of the digit change depending on its position in a decimal number? Circle the correct answer.*

  Yes *No*

Problem 13.16

- *What is the value of the bold-face digit* **1** *in the binary number* 10**1**0*?*

  The value is _____ .

- *What is the value of the bold-face digit* **1** *in the binary number* 11000**1***?*

  The value is _____ .

- *What is the value of the bold-face digit* **1** *in the binary number* 10**1**10*?*

The value is _____ .

- *Does the value of the digit change depending on its position in a binary number? Circle the correct answer.*

Yes *No*

A numeral system where the value of a digit depends on its position in a number is called *place-value*. The decimal and binary systems are place-value.

Problem 13.17

- *What is the value of the bold-face digit* **I** *in the Roman number X***I***?*

The value is _____ .

- *What is the value of the bold-face digit* **I** *in the Roman number* **I***II?*

The value is _____ .

- *What is the value of the bold-face digit* **D** *in the Roman number C* **D***?*

The value is _____ .

- *What is the value of the bold-face digit* **D** *in the Roman number* **D***C?*

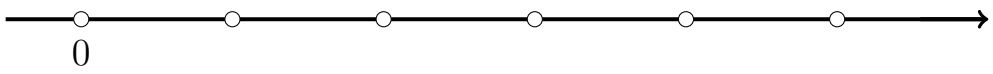
The value is _____ .

- *Does the value of a digit change depending on its position in a Roman number? Circle the correct answer.*

Yes *No*

Question 13.7 *Is the Roman numeral system place-value? Why or why not?*

Problem 13.18 *The left-most mark on the binary number line below is zero. Mark the rest of the line. To save space, do not write B in front of the numbers.*

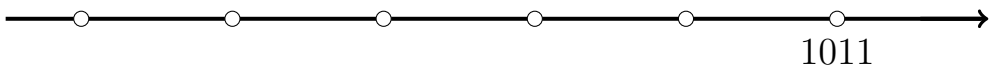


Problem 13.19 *You have nine coins that look the same. One of them is a fake. You do not know whether the fake is lighter or heavier than the real coins. Given a balance scale with no weights, find the fake coin in no more than three weighings.*

Homework

Finish all the problems unfinished in class. Teach your family to count in binaries.

Problem 13.20 *The right-most mark on the binary number line below is B* 1011. *Mark the rest of the line. To save space, do not write B in front of the numbers.*



Problem 13.21 *Find the binary notation for the number* 146.

studied number	*basic binary numbers*							
	128	**64**	**32**	**16**	**8**	**4**	**2**	**1**
146								

$146 = B$ _____

Problem 13.22 *Break the number* 146 *into a sum of basic decimal numbers. Write the coefficients in the table below. Then make the number on the abacus and check that it shows the same.*

studied number	*basic decimal numbers*				
	10000	**1000**	**100**	**10**	**1**
146					

Question 13.8 *What is in common and what is different between the decompositions of the number* 146 *in problems 13.21 and 13.22?*

Name all the reasons to call the binary system binary. Name all the reason to call the decimal system decimal.

Question 13.9 *What is in common and what is the difference between the binary and decimal systems?*

Problem 13.23

- *What is the value of the bold-face digit* **9** *in the decimal number* 201**9***?*

  The value is _____ .

- *What is the value of the bold-face digit* **9** *in the decimal number* 2**9**5*?*

  The value is _____ .

- *What is the value of the bold-face digit* **9** *in the decimal number* 9**9**999*?*

The value is _____ .

- *Does the value of the digit change depending on its position in a decimal number? Circle the correct answer.*

Yes *No*

Problem 13.24

- *What is the value of the bold-face digit* **1** *in the binary number* 10**1**00*?*

The value is _____ .

- *What is the value of the bold-face digit* **1** *in the binary number* 1**1**1110*?*

The value is _____ .

- *What is the value of the bold-face digit* **1** *in the binary number* 101**1***?*

The value is _____ .

- *Does the value of the digit change depending on its position in a binary number? Circle the correct answer.*

Yes *No*

Question 13.10 *What numeral systems do we call place-value?*

Question 13.11 *What place-value systems do you know?*

Problem 13.25

- *What is the value of the bold-face digit* **M** *in the Roman number* **M***MXIX?*

The value is _____ .

- *What is the value of the bold-face digit* **M** *in the Roman number MM* **M***?*

The value is _____ .

- *Does the value of the Roman numeral digit change depending on its position in a number? Circle the correct answer.*

Yes *No*

Question 13.12 *Is the Roman numeral system place-value? Why or why not?*