

# Lesson 3: Weighings, Invariants and Geometric Constructions

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## 1 From Last Week

### Problem 3.

A square is split into 100 rectangles using 9 vertical and 9 horizontal lines. Exactly 9 of those rectangles are squares – show that two of those squares have the same side length.

### Problem 4.

Suppose you have five positive integers, and you computed all ten of their pairwise sums. Is it possible that the ten pairwise sums all have different last digits?

## 2 New Problems

### Problem 1.

You are given four coins with weights  $1g$ ,  $2g$ ,  $3g$  and  $4g$ , but you do not know which ones are which. Is it possible to determine the weight of each coin using a balance scale at most 4 times?

### Problem 2.

All four corners of an  $8 \times 8$  board are colored black, and the rest of the squares are white. You are allowed to change the colors of all squares in a single row or column. Show that it is impossible to use such operations to make all the squares white.

### Problem 3.

Consider the numbers 1 through 6 placed on a circle in order. It is allowed to add one to three consecutive numbers, or to subtract one from three numbers no two of which are adjacent. Is it possible to use these operations to make all six numbers equal?

### Problem 4.

Show how to construct the center of a given circle.

### Problem 5.

Let  $P$  be a point outside of a given circle. Show how to construct the two tangent lines from  $P$  to the circle.