Modular Arithmetic Problems

Problem 1) How many different residue classes are there, $\mod 4$? ... $\mod 10$? ... $\mod m$?

Problem 2) Prove: if I have $m+1$ integers, at least two of them will be congruent $\mod m$.

Problem 3) With a partner, test out the theorems I've written down... 10 points if you can stump a grad student.

Problem 4) Compute:

(i) $22^2 - 1^2 \mod 23$; (ii) $22^4 - 1^4 \mod 23$;
(iii) $22^{20} - 1^{20} \mod 23$; (iv) $22^{100} - 1^{100} \mod 23$;
(v) $45^{100} - 24^{100} \mod 23$; (vi) $2345^{100} - 2324^{100} \mod 23$.

Problem 4) Prove the first theorem. (The first time I did this, I made up a few examples just to see how (if?) things worked. I don't remember *exactly* which numbers I tested, but I wouldn't be surprised if 23 appeared.)

Problem 5) Prove the second theorem. Why is the first theorem a special case of the second?