Remainder Problems

Problem 1) Given any integer, k, we saw that we can express

$$k = p \times 10 + r,\ 0 \leq r < 10.$$

Suppose I can find another pair of integers, q, s such that I can express

$$k = q \times 10 + s,\ 0 \leq s < 10.$$

What can we saw about p, r vs. q, s? (Hint: do an example!)

Problem 2) Given any integer, k, we can choose p, r that allow us to express

$$k = p \times 23 + r,\ 0 \leq r < 23.$$

Using the previous problem as a model, what can we say about our "choice" of p, r?

You might try at least one of the previous problems before you give this one a shot:
Problem 3) Fix an arbitrary (random) integer, x. Prove the following:
Given any integer, k, we can choose *unique* integers p, r such that we can express

$$k = px + r,\ 0 \leq r < x.$$

Problem 4) Just to get a feel for what's going on, pick an integer, x, and have a partner pick a different integer, y. Between the two of you, find p, r and q, s (where $0 \leq r < p$, $0 \leq s < q$) that allow you to express

$$y = px + r,\ \text{and}\ x = qy + s.$$

Once you've done this, you might substitute:

$$y = p(qy + s) + r,\ \text{or}\ x = q(px + r) + s.$$

What happens? Why?