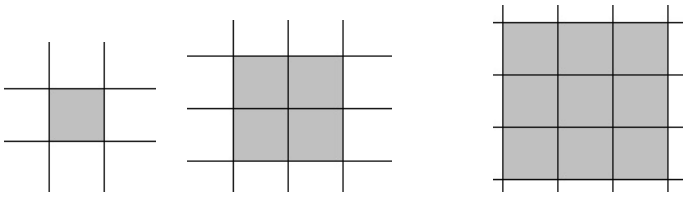
Similar Shapes and Gnomons

October 10, 2010

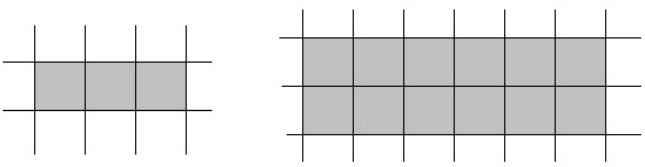
1. Similar Shapes

For now, we will say two shapes are *similar* if one shape is a "magnified" version of another.

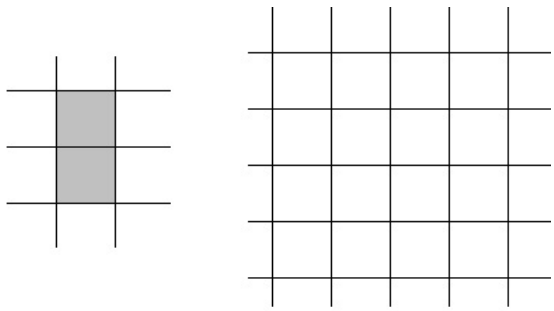
1. In the picture below, the square on the left is 1 box wide and 1 box high (1x1). The square in the middle is *similar* to the first square because it still looks like the original square, magnified twice. It is 2 boxes wide and 2 boxes high (2x2). The square on the right is also *similar* to the first because it is the original square, magnified 3 times (3x3). All three squares are *similar*, then.



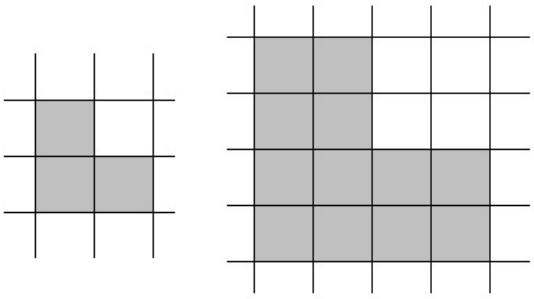
2. Below, the rectangle on the left is 3 boxes wide and 1 box high (3x1). The rectangle to the right is *similar* to the one on the left. It is the same rectangle, magnified twice. Both the width and the length of the rectangle are twice those in the original rectangle on the left: it is 6 boxes wide and 2 boxes high.



3. Below is a picture of a rectangle 1 box wide and 2 boxes high. Draw and shade in a rectangle on the right that is *similar* to the original rectangle (magnified twice).



4. We can also find *similar* shapes that are not just squares or rectangles. Consider the two L-shapes below:

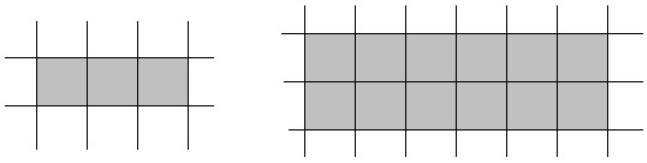


   (a) What are some things that the two shapes have in common?

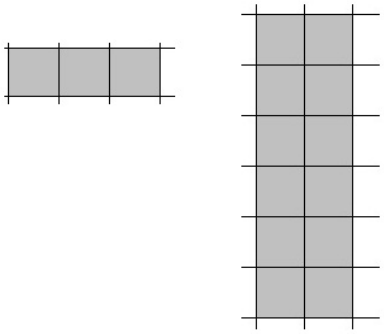
   (b) What are some differences between the shapes?

   (c) Do you think they similar? Why or why not?

5. One important thing to note with *similar* shapes is that the orientation of the shapes (the way they are rotated) does not matter. Remember this example from above?



The two rectangles below are also similar:

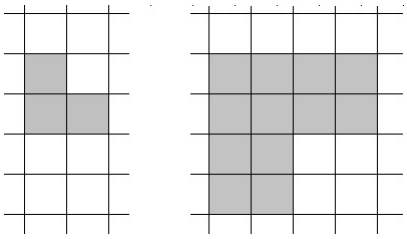


(a) How are the first pair of rectangles like the second pair?

(b) How are the two pairs of rectangles different?

(c) How are the two larger rectangles alike? Do you think they are also *similar* to each other?

6. Like with rectangles, rotation does not affect whether or not two shapes are similar:



   (a) What are some things that the two shapes have in common?

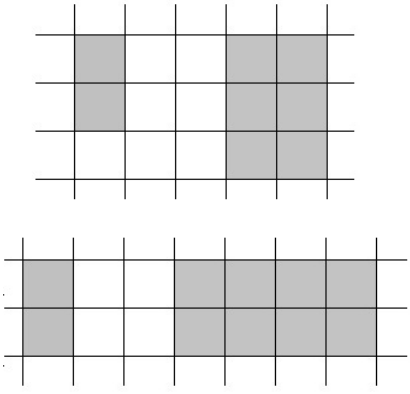
   (b) What are some differences between the shapes?

   (c) Do you think they are *similar*? Why or why not?

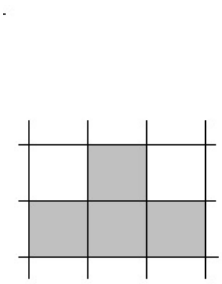
7. When trying to decide if two shapes are *similar* by our definition, think about the following questions:

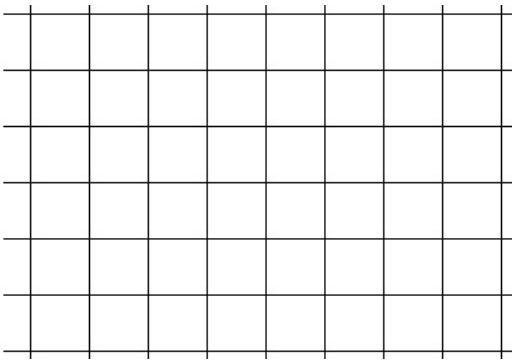
   (a) Does the larger shape look like a magnified version of the smaller?
   (b) If the width of the shape is magnified by a certain factor, is the height also magnified by that same factor?
   (c) If they could be rotated, would they still look similar?

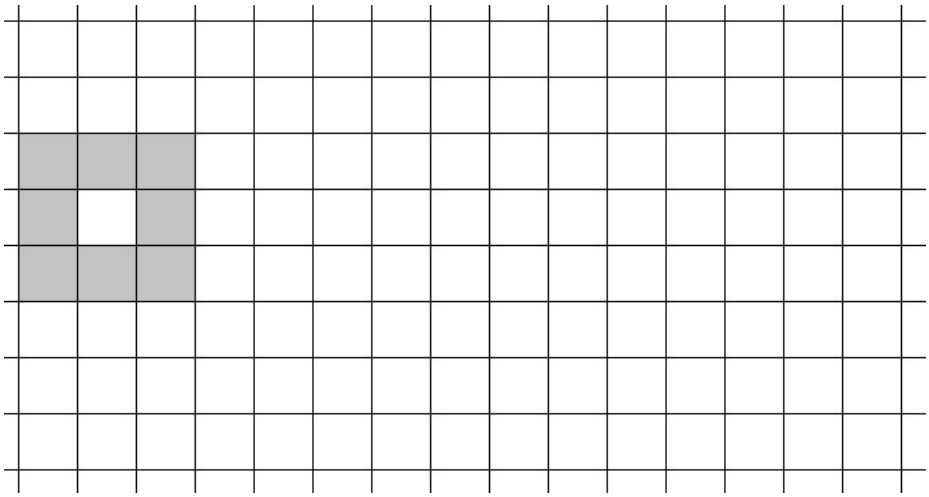
   Are either of the following pairs of rectangles similar?



8. Use the grid to the right of the shapes to draw a *similar* shape that looks like the original, magnified twice.





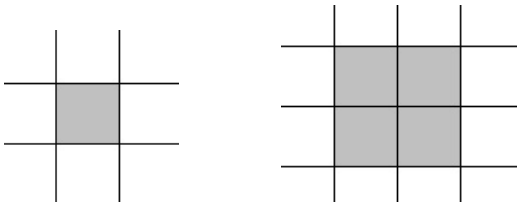


2. Gnomons

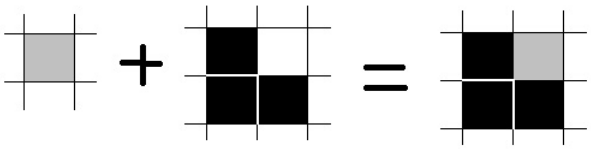
What is a "gnomon?"

A gnomon is a piece that you can attach to another shape to create a shape that is *similar* to the original.

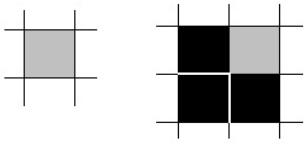
1. Consider two similar squares from the first page:



What piece could we attach to the square on the left to make the square on the right? Below is one possibility:

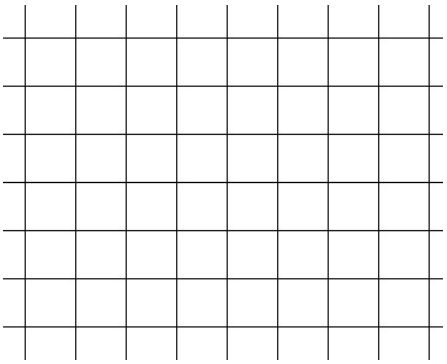


When the dark-colored L-shape is attached to the original 1x1 square, the new shape is a square that is *similar* to the original square.

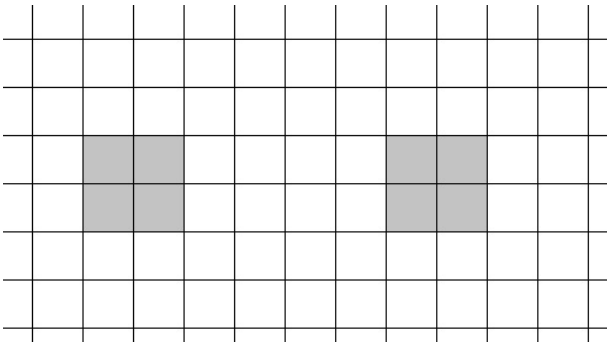


That L-shape  is therefore known as a *gnomon* for the square.

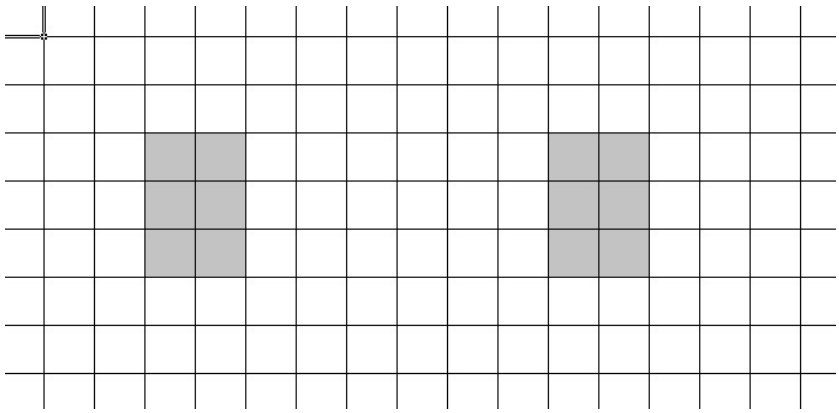
(a) Find another *gnomon* that makes a 1x1 square into a 2x2 square:



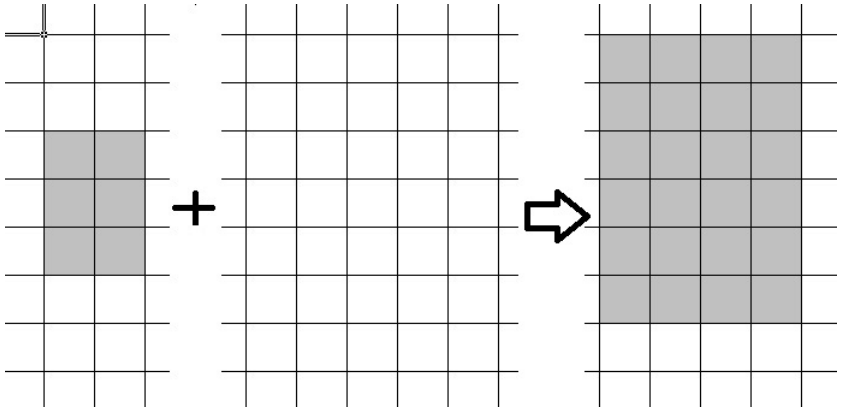
(b) We learned above that a 2x2 square and 3x3 square are also similar. Find the piece (gnomon) that can be added to the 2x2 square to make it a 3x3 square:



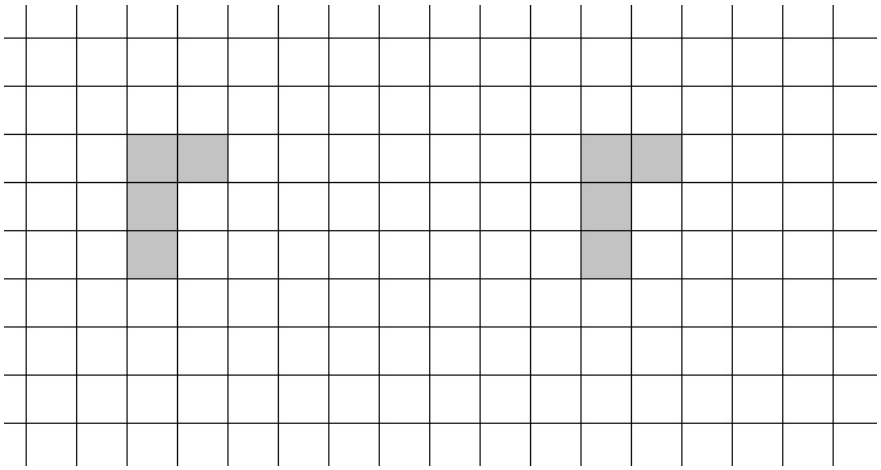
2. Find a gnomon for this rectangle that magnifies it by a factor of two (the resulting *similar* rectangle should be twice as big as the original). You can use the grids below.



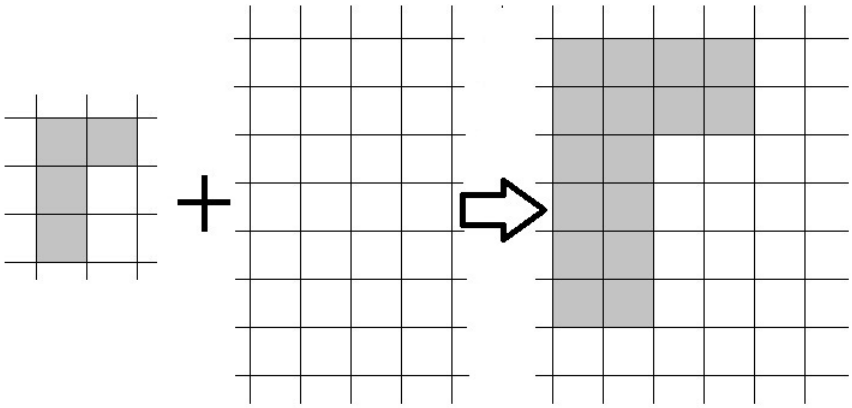
(a) So what does the gnomon look like? Draw it in the blank spot below:



3. Find a gnomon that, when attached to the shape below, creates a *similar* shape that is the original magnified by a factor of 2.

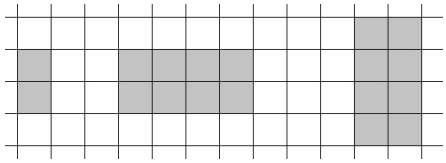


(a) What does the gnomon look like? Fill in the blank spot below

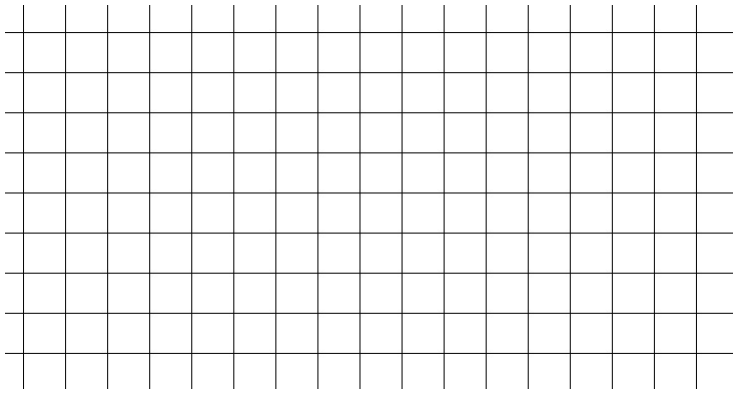


(b) Do you think there are any other gnomons for this shape?

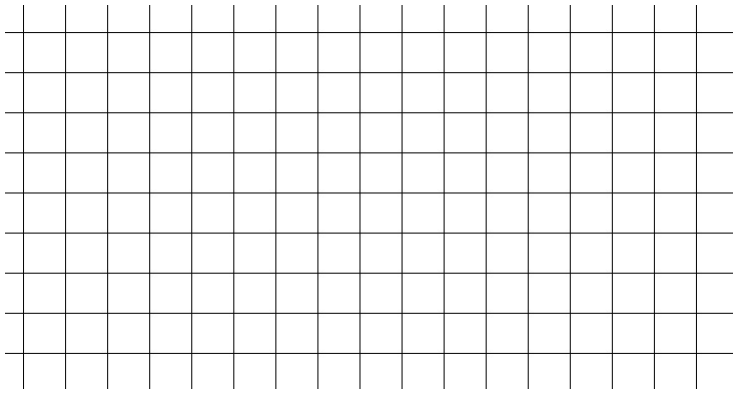
4. Remember that two shapes can still be *similar* if they are rotated differently. The three rectangles below are all similar:



(a) Find a gnomon that will make the rectangle on the left into the rectangle in the middle



(b) Find a gnomon that will make the rectangle on the left into the rectangle on the right

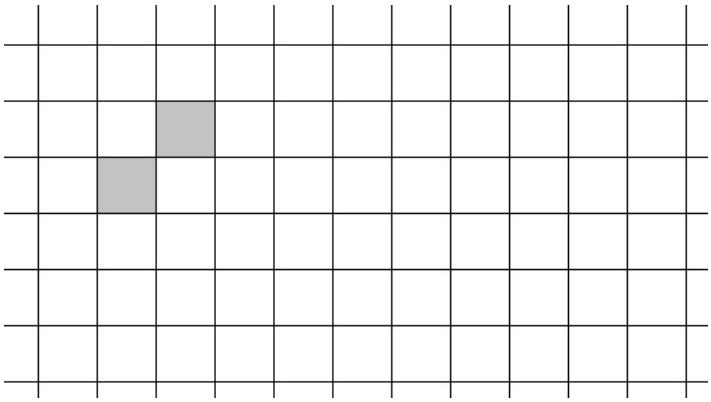


(c) Do the two gnomons look the same?

(d) What do the gnomons have in common (Hint: How many boxes are they made up of?)

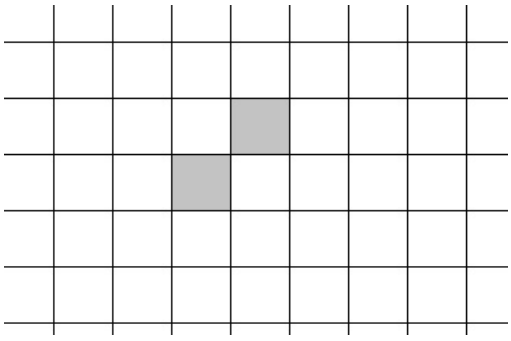
3. Disconnected Gnomons

1. Next to the shape below, draw a similar shape that is the original magnified by a factor of two:



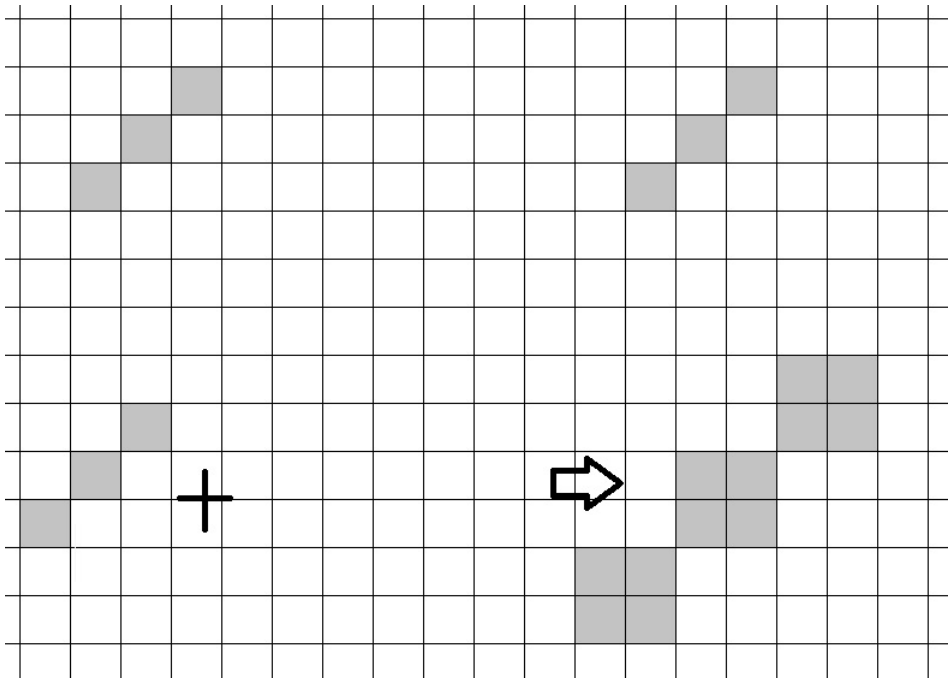
(a) Looking at the two shapes above, can you find one connected piece that will give a *similar* shape when attached to the original?

(b) What if you are allowed to add two pieces? Draw them in below. (The two added pieces may be different)

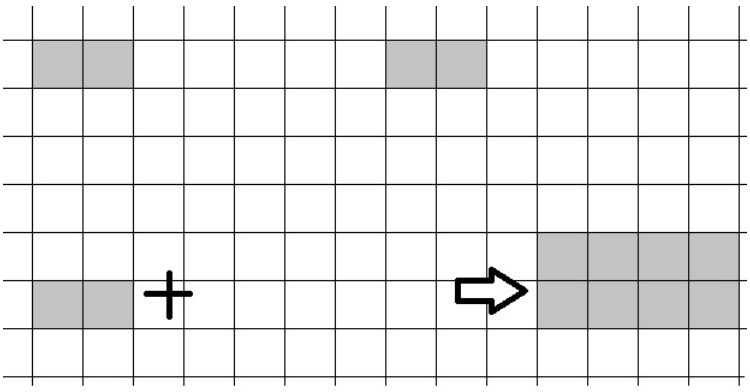


2. As it turns out, some shapes do not have a connected gnomon (a gnomon which consists of a single piece). In these cases, though, sometimes, we can still make shapes *similar* to the original by adding a few additional pieces. We will call the collection of pieces we add to make a *similar* shape a *disconnected gnomon*.

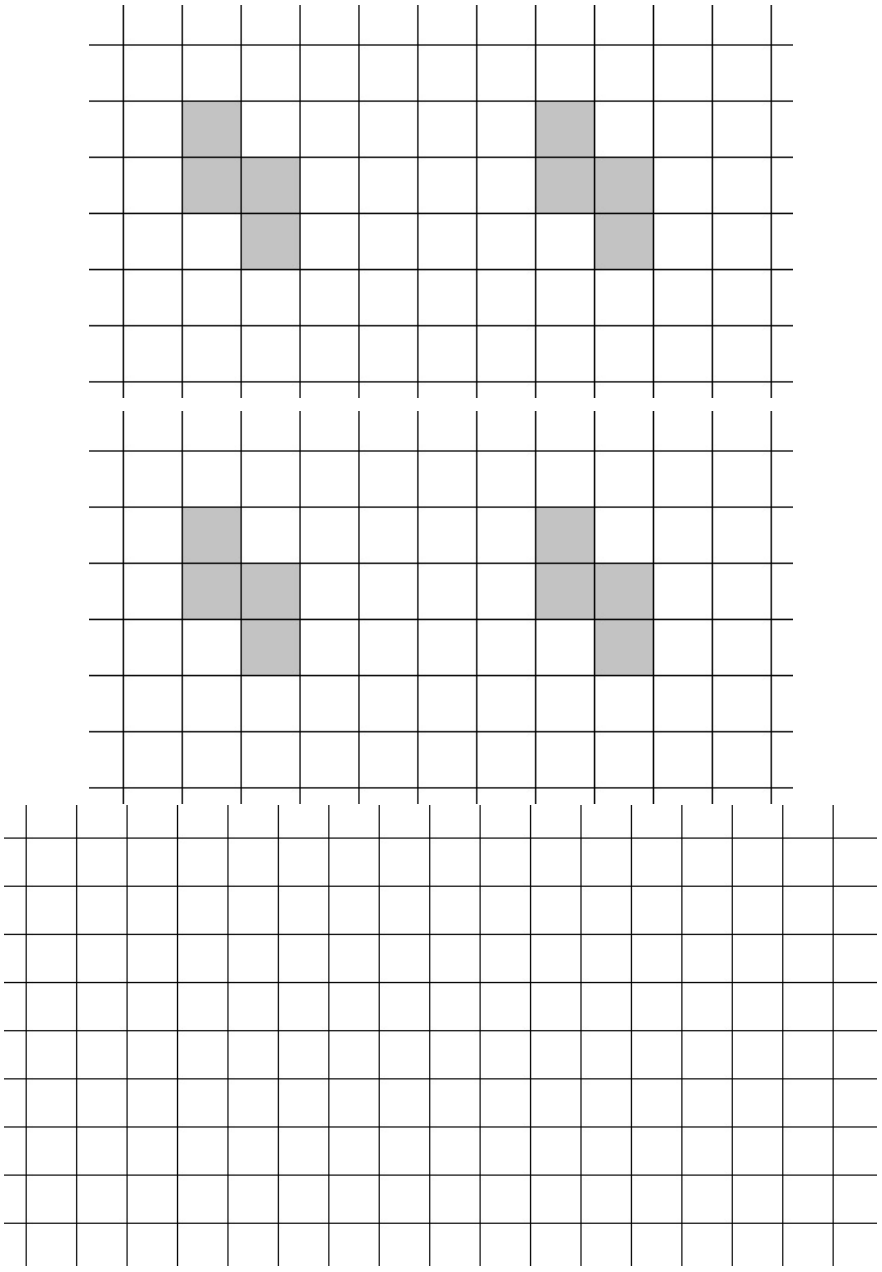
   (a) Does this shape have a connected or disconnected gnomon?



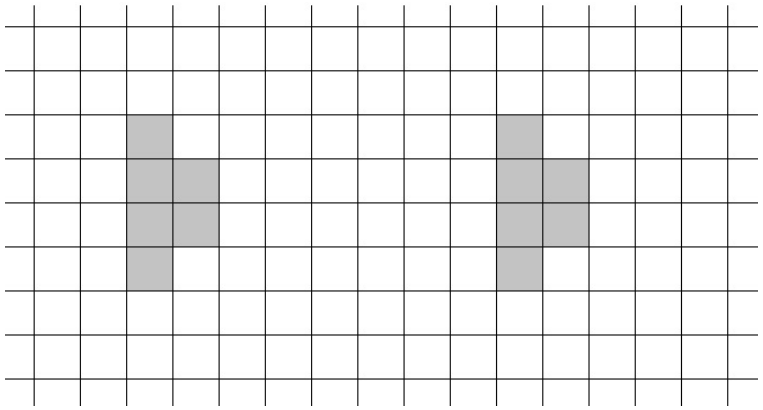
   (b) Does this shape have a connected or disconnected gnomon?

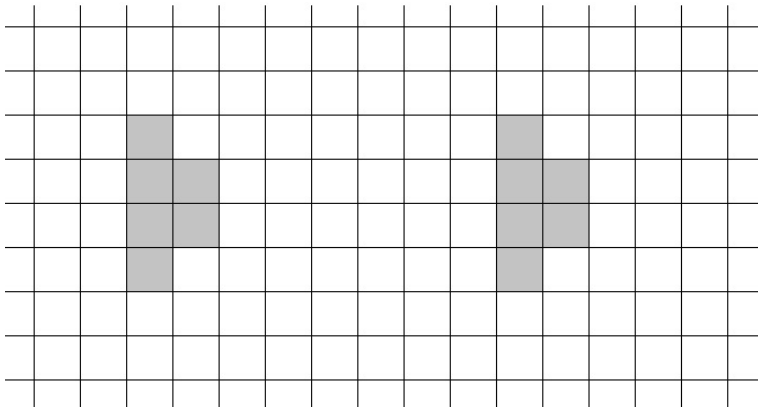


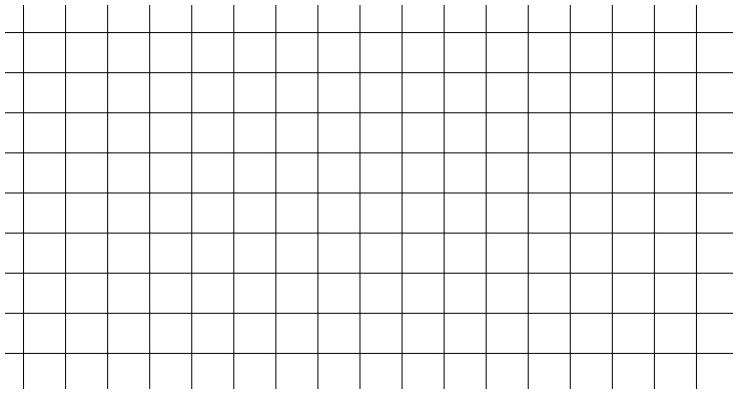
3. Find two different gnomons for the shape below:



4. Sometimes, a shape can have both a connected and a disconnected gnomon. Can you find one connected gnomon and one disconnected gnomon for the shape below?

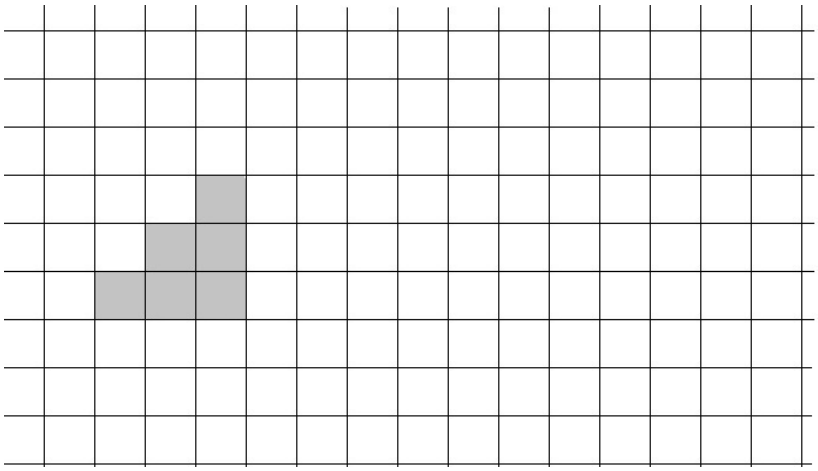






5. It is important to notice that while many shapes do have *gnomons* or *disconnected gnomons*, some shapes do not. For those shapes, it is impossible to create a *similar* shape simply by adding on to the original.

   (a) Suppose we magnify the following shape by a factor of 2. Draw the *similar* shape to the right of the original.

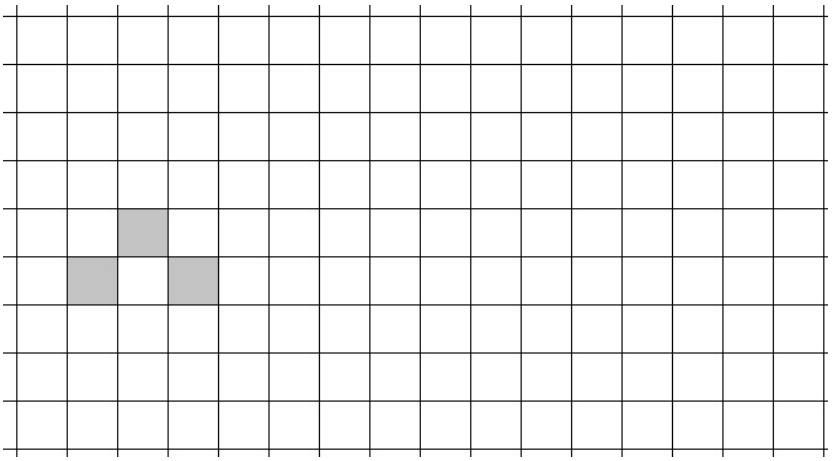


      i. Compare the original and magnified shapes. Does this shape have a gnomon?

         A. If the shape has a gnomon, draw the gnomon around the shape above. Is the gnomon you found connected or disconnected?

         B. If it does not have a gnomon, explain why?

6. Suppose we magnify the following shape by a factor of 2. Draw the *similar* shape to the right of the original.



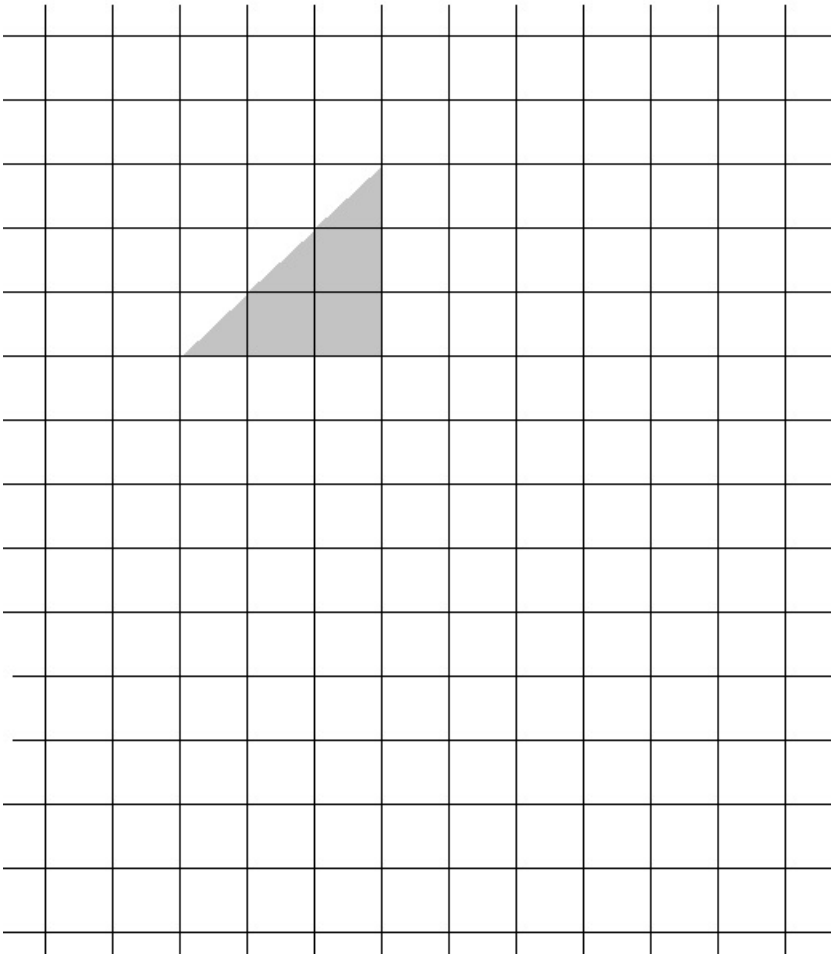
(a) Compare the original and magnified shapes. Does this shape have a gnomon?

i. If the shape has a gnomon, draw the gnomon around the shape above. Is the gnomon you found connected or disconnected?

A. If it does not have a gnomon, explain why?

4. Challenge

1. Find two gnomons for the following triangle, which has two equal sides? (If it seems confusing, remember that this triangle is exactly one half of a square cut diagnally.)



5. Homework

Come next week with a shape (not a square or rectangle) with a connected gnomon that, when connected to the original shape, *triples* the size of the shape. Draw the original shape, gnomon, and the larger *similar* shape separately. It may help to use paper with a grid on it, such as graph paper.