Homework 7: Stable Matching

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Problem 1.
Consider the same setup as in the stable matching problem discussed during the lesson, but now some man-woman pairs are explicitly forbidden beforehand. In this more general setting, we say that a matching S is stable if it does not exhibit any of the following types of instability.

i) There are two pairs (m, w) and (m', w') in S with the property that (m, w') is not forbidden, m prefers w' to w, and w' prefers m to m'. (The usual kind of instability.)

ii) There is a pair (m, w) in S, and a man m', so that m' is not part of any pair in the matching, (m', w) is not forbidden , and w prefers m' to m. (A single man is more desirable and not forbidden.)

iii) There is a pair (m, w) in S, and a woman w', so that w' is not part of any pair in the matching, (m, w') is not forbidden, and m prefers w' to w. (A single woman is more desirable and not forbidden.)

iv) There is a man m and a woman w, neither of whom is part of any pair in the matching, so that (m, w) is not forbidden. (There are two single people with nothing preventing them from getting married to each other.)

Notice that in this context, the matching need not be perfect, meaning that in the end there could be unmatched men and/or women. Your goal in this homework will be to modify Gale-Shapley algorithm for this generalized situation. Here is what you need to do:

a) Formulate the version of Gale-Shapley algorithm for this problem. *Hint: it will be exactly the same as the normal Gale-Shapley, except skipping forbidden pairs.*

b) Show that your algorithm from a) results in a stable matching in the sense of the four conditions outlines above (that is, none of them apply to the resulting matching).