Alternative solution to L4.3

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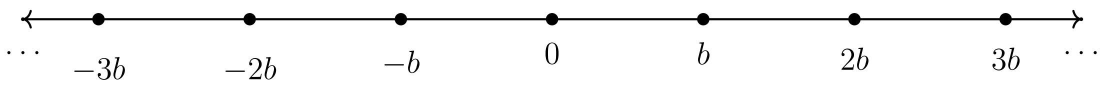
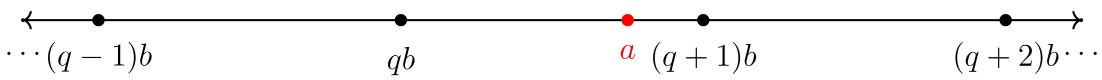
November 5, 2017

The solution above is the geometric interpretation of the algebraic solution you already have – perhaps some of you will find this interpretation more clear.

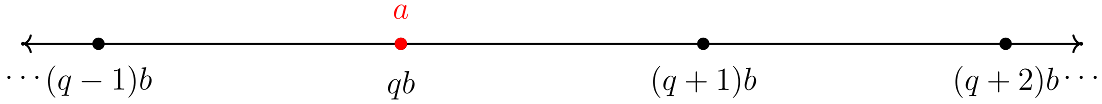
Problem 3.
We will give the solution straight for part b) of the problem. First suppose $b > 0$. Then let us mark the points $0, b, 2b, 3b, \ldots$ on the coordinate line. And the same for negative multiples of b: $-b, -2b, -3b$ and so on.



If we now represent a as a point on the coordinate line, it will fall between some pair of marked points, lets call them qb and $(q + 1)b$.



If a falls directly on a marked point, we will call that point qb:



Now let us set $r = a - qb$. Since a is between qb and $qb + b$ we know that $r = a - qb < qb + b - qb = b$, so $r < b$ and clearly $r \geq 0$. Thus these r and q

work, which concludes the case $b > 0$.

If $b < 0$, we can use the previous case: if we find q and $0 \leq r < |b|$ such that $a = q(-b) + r$, then it also holds that $a = (-q)b + r$ which is the desired formula.

The argument we presented proves that q, b exist. As far as the uniqueness goes, one can either follow the argument in part a) of the original algebraic solution, or consider the following geometric viewpoint: if one chooses $q'b$ to be any point with $q' > q$ where q is the one we chose, then $q'b$ will be to the right of a and $r' = a - q'b$ will have to be negative. If we choose $q' < q$, then the point $q'b$ will be at least length b far from a to the left, and so $r' = a - q'b > b$ which is also prohibited. So the choices of q and r we made were in fact forced, and thus unique. As for the case $b < 0$, uniqueness follows from the uniqueness of the remainder when divided by $-b$: if q, r are unique solutions for the equation $a = q(-b) + r$ with $0 \leq r < b$, then the ones for $a = qb + r$ are unique as well since they differ only by changing the sign of q.