Homework 3 Problem 2 Solution.

Mingxi Sun

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Problem 2

Color the infinite board diagonally with 3 colors. Initially, there is a rectangle with n columns and m rows occupied by pawns. Since n is divisible by 3, each row can be divided into $n/3$ 1×3 rectangles, each rectangle having one of each color. Doing so for each row, the whole $m \times n$ rectangle can be divided into $mn/3$ 1×3 rectangles. So there are $\frac{mn}{3}$ pawns standing on each color i.e the number of pawns of each color is equal. Each legal move will increase the number of pawns of one color by 1 and decrease the number of pawns of the other two colors by 1. Suppose before this move there are k pawns of each color.

Case 1: k is even: $k-1$, $k+1$ will both be odd. So the number of pawns of each color after this move will be odd.

Case 2: k is odd: $k-1$, $k+1$ will both be even. So the number of pawns of each color after this move will be even.

Therefore after every move, the number of pawns of each color will always have the same parity (even or odd). But if there is only one pawn left at the end, the number of pawns of one color will be odd since 1 is odd and the number of pawns of the other two colors will both be even since 0 is even, which is a contradiction.