Fun and Games on a Chess Board

Names of squares on the chess board

Color the following squares on the chessboard below:

c3, c4, c5, c6, d5, e4, f3, f4, f5, f6

What letter do these squares form together?
II How many squares are there on a chessboard?

A chessboard itself is a square with side 8.

1. The number of $1 \times 1$ squares on the chess board is $8^2$.

2. What about bigger squares?
   Let’s first count squares of size $2 \times 2$:
   
   Idea: Instead of counting $2 \times 2$ squares, we will count the small $1 \times 1$ squares which can serve as the left lower corners of the $2 \times 2$ squares that fit on the chessboard.

   First, shade the left lower corner of the $2 \times 2$ square above.
For each of the squares below, decide if it can be a left lower corner of a $2 \times 2$ square:

(a) square c3  
Yes  No

(b) square g6  
Yes  No

(c) square f8  
Yes  No

(d) square h2  
Yes  No

Now color all $1 \times 1$ squares that can serve as the left lower corners of a $2 \times 2$ square:

How many $2 \times 2$ squares can you fit onto a chessboard?
3. For each of the squares below, decide if it can be a left lower corner of a $3 \times 3$ square:

(a) square e6  Yes  No

(b) square g3  Yes  No

(c) square a7  Yes  No

(d) square f6  Yes  No

Now color all $1 \times 1$ squares that can serve as the left lower corners of a $3 \times 3$ square:

![Chessboard diagram]

How many $3 \times 3$ squares can you fit onto a chessboard?
Now you can fill out the table below:

<table>
<thead>
<tr>
<th>Type of Square</th>
<th>Number of such squares</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Single Square" /></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="4 Squares" /></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="9 Squares" /></td>
<td></td>
</tr>
</tbody>
</table>
Homework

Count the number of $4 \times 4$, $5 \times 5$, $6 \times 6$ and $7 \times 7$ squares on the chessboard in the same way. In each case, use a chessboard picture to shade all the $1 \times 1$ squares that can be left lower corners of the bigger squares that fit completely onto the chessboard.

Now color all $1 \times 1$ squares that can serve as the left lower corners of a $4 \times 4$ square:
Color all $1 \times 1$ squares that can serve as the left lower corners of a $5 \times 5$ square:

Color all $1 \times 1$ squares that can serve as the left lower corners of a $6 \times 6$ square:
Color all $1 \times 1$ squares that can serve as the left lower corners of a $7 \times 7$ square:

![7x7 Grid](image)

Color all $1 \times 1$ squares that can serve as the left lower corners of a $8 \times 8$ square:

![8x8 Grid](image)