1. There are 20 students in a Math Circle. 10 students like apples and 15 students like pears.

   (a) Show that there are some students who like both apples and pears. (Hint: Assume the opposite and try to get a contradiction)

   (b) Is it possible to determine if there any students who do not like apples and do not like pears? If yes, explain how you can do it. If no, demonstrate by giving examples.

   (c) Assume that each student likes at least one of the fruits. (This means that each student like either apples, or pears, or both). How many students like both pears and apples?

2. The same Math Circle (with 20 students) forms a soccer team and a basketball team. Every student signs up for at least one team:
• 12 students play only soccer;
• 2 students play both soccer and basketball;

How many students play basketball only?

3. In the same Math Circle, each student takes a foreign language class (Spanish or French):
   • The French class has 12 students;
   • 5 students are enrolled in both French and Spanish class

(a) How many students are enrolled only in the French class?

(b) How many students are enrolled only in the Spanish class?
4. In the same Math Circle, all students traveled to either San Francisco or San Diego:

- 12 students visited San Francisco;
- 14 students visited San Diego;

(a) How many students visited both destinations?
(b) How many students visited just San Francisco?
(c) How many visited just San Diego?
5. Divide each of the shapes below into two shapes which are the same both in size and in shape. (Two copies of each shape are provided. Practice on the top copies; present your answer on the bottom copies)

6. Olga collected some bugs and spiders to show to math circle students and put them into a jar. All together, there are 8 creatures in the jar. Together they have 54 legs. (Note: spiders have 8 legs and bugs have 6 legs).
(a) How many legs would there be if all the creatures were bugs?

(b) How many bugs would you have to replace with spiders so that there are 54 legs?

(c) How many bugs are there and how many spiders are there? Check your answer

7. Jake and Sam are friends that like to joke. Today, they decided that one of them going to answer truthfully to all questions and the other is going to lie.

(a) Come up with a question to ask both of them that will tell you who is telling the truth and who is lying.

(b) Now, assume that Jake and Sam are twins and you cannot tell them apart. Come up with another question that will tell you who is telling the truth and who is lying.
8. Make up your own problem on Venn Diagrams. At home, write your problem down in complete sentences on an index card. Next week, you will trade your index card with a partner and you will solve your partner’s problem.