Quiz 1

Name:

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To receive credit, please show all your work. Good luck!

PROBLEM 1. Compute the following limits using the limit laws: (a)

$$\lim_{n \to \infty} \frac{3n^4 + (-1)^n n^3 - 1}{n^4} = \lim_{n \to \infty} \left(3 + \frac{(-1)^n}{n} - \frac{1}{n^4}\right) = 3 + 0 - 0 = 3.$$

(b)

$$\lim_{n \to \infty} \frac{1}{2^n} - \frac{1}{n^2} = 0 - 0 = 0$$

PROBLEM 2. Let $a_n = \frac{n+3}{n-1}$ (a) Find $\lim_{n\to\infty} a_n$. $\lim_{n\to\infty} \frac{n+3}{n-1} = \lim_{n\to\infty} \frac{(n-1)+4}{n-1} = 1 + \lim_{n\to\infty} \frac{4}{n-1} = 1$. (b) Let a be the limit that you have found in part (a). Let $\varepsilon = 0.4$. Find N > 0 such that for all n > N we have $|a_n - a| < 0.4$.

The limit found in part (a) is a = 1.

$$\begin{array}{c|c} \frac{n+3}{n-1} - 1 \\ \left| \begin{array}{c} 4 \\ \frac{4}{n-1} \\ \end{array} \right| < 0.4 \\ \frac{4}{0.4} < n-1 \\ 11 < n \end{array}$$

Take N = 11. Then for any n > 11 we have $|a_n - 1| < 0.4$.