PRACTICE PROBLEMS FOR MIDTERM 1

(MATH 3A)

1. Use the limit laws to find limits of the following sequences:
(a) $\lim_{n\to\infty} \frac{2n-1}{n}$;
(b) $\lim_{n\to\infty} \frac{3^{-n}-2^{-n}}{n}$;
(c) $\lim_{n\to\infty} \frac{n+1}{n^2-1}$;
2. Use the formal definition of limits to show that $\lim_{n\to\infty} a_n = 0$. That is, for every $\varepsilon > 0$ find $N > 0$ so that $|a_n - a| < \varepsilon$ for all $n > N$:
(a) $\lim_{n\to\infty} \frac{1}{n^2+1} = 0$;
(b) $\lim_{n\to\infty} \frac{1}{\sqrt{n+3}} = 0$;
(c) $\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$;
3. Find the following limits of functions:
(a) $\lim_{x\to 2} \frac{x^2e^x+2}{4}$;
(b) $\lim_{x\to 1} \frac{x^2-1}{x-1}$;
(c) $\lim_{x\to 0} \frac{x-\sqrt{x^2+25}}{2x^2}$;
(d) $\lim_{x\to 0} \frac{\sqrt{2-x}-\sqrt{2}}{2x}$;
(e) $\lim_{x\to 2+}[x+1]$ and $\lim_{x\to 2-}[x-1]$;
(f) $\lim_{x\to\infty} \frac{e^{-x}}{1-e^{-x}}$;
(g) $\lim_{x\to-\infty} \frac{2x+x^2}{-x-1}$;
(h) $\lim_{x\to\infty} \frac{3x^4-x^3+1}{x^4+2x^2}$;
(i) $\lim_{x\to-\infty} \frac{e^x}{3x^2-x}$;
(j) $\lim_{x\to\infty} \frac{xe^{-x}}{3x+1}$;
4. Find the values of $x \in \mathbb{R}$ for which the given functions are continuous:
(a) $f(x) = \ln \frac{x+1}{x-1}$;
(b) $f(x) = \ln(x^2-4)$;
(c) $f(x) = e^{\tan(x)}$;
(d) $fx) = \tan(2x+1)$;
5. Let $f(x) = \begin{cases} ax^2 & , \ x > 2 \\ x+6 & , \ x \leq 2 \end{cases}$. Graph the function $f(x)$ when $a = 1$. Is this function continuous for $a = 1$? How must you choose a so that $f(x)$ is continuous for all $x \in (-\infty, \infty)$?

6. Let $N(t) = \frac{10^4}{10+e^{-5t}}$ be the equation describing the population growth. What is the initial size of the population ($N(0)$)? What is the carrying capacity K?
7. Evaluate the following trigonometric limits:
   (a) $\lim_{x\to 0} \frac{1-\cos(2x)}{x}$;
   (b) $\lim_{x\to 0} \frac{\sin(2x)}{x(1-x)}$;
   (c) $\lim_{x\to 0} \frac{\sin(7x)}{x}$;
   (d) $\lim_{x\to 0} x^2 \cos \frac{1}{x}$;
   (e) $\lim_{x\to 0} \frac{\sin(\sin(x))}{x^2}$;
   (f) $\lim_{x\to 0} \frac{\sin(x^3)}{\sin(\sqrt{x})}$;
8. From the point of view of the intermediate value theorem, explain why the equation $y = x^2 - \pi$ has two roots.
9. Using the intermediate value theorem, find an interval on which the equation $e^{2x} = x^2$ has a solution.
10. Show that any polynomial of an odd degree (that is, degree $1, 3, 5$,etc.) has at least one root. What about polynomials of even degrees ($2, 4$,etc.)?