MATH 3A
SECOND MIDTERM EXAMINATION

February 27th, 2006

Please show your work. Except on the multiple choice problems, you will receive little or no credit for a correct answer to a problem which is not accompanied by sufficient explanations. If you have a question about any particular problem, please raise your hand and one of the proctors will come and talk to you. At the completion of the exam, please hand the exam booklet to your TA. If you have any questions about the grading of the exam, please see the instructor *within 15 calendar days of the examination.*

Name:______________________________ Section:____________________

#1	#2	#3	#4	#5	Total

R

Problem 1. Find the derivative of the function:

$$y = 2x^{1/2} + 3x^{-2/3} + \ln(e^x + 3) + \sin(\sqrt{x}).$$

SOLUTION.

$$y' = x^{-1/2} - 2x^{-5/3} + \frac{1}{e^x + 3} e^x + \frac{1}{2\sqrt{x}} \cos(\sqrt{x}).$$

Problem 2. Find $\frac{dy}{dx}$ by implpicit differentiation $\sqrt{xy} = x^2 + 1$.

SOLUTION. Differentiating the given equality, we obtain

$$\begin{aligned}
\frac{1}{2\sqrt{xy}} \cdot (y + xy') &= 2x \\
y + xy' &= 4x\sqrt{xy} \\
y' &= 4\sqrt{xy} - \frac{y}{x}, \qquad \text{this can also be written as:} \\
y' &= 4(x^2+1) - \frac{y}{x}.
\end{aligned}$$

Problem 3. Let $f(x) = x - \sqrt{x}$, $x \geq 0$. Find $\frac{d}{dx} f^{-1}(2)$.

SOLUTION. $x - \sqrt{x} = 2$, $x \geq 0$ implies $x = 4$. Thus, $f(4) = 2$ and $f^{-1}(2) = 4$. We have

$$\frac{d}{dx} f^{-1}(2) = \frac{1}{f'(4)}.$$

Compute the derivative of $f(x) : f'(x) = 1 - \frac{1}{2\sqrt{x}}$. Thus, $f'(4) = 1 - \frac{1}{4} = \frac{3}{4}$. We obtain that $\frac{d}{dx} f^{-1}(0) = 4/3$.

Problem 4. Compute the derivative of the function $y = x^{\sin(x)}$.

SOLUTION. Take the logarithm of both sides:

$$\begin{aligned}
\ln(y) &= \sin x \cdot \ln x \\
\frac{y'}{y} &= \cos x \cdot \ln x + \sin x \cdot \frac{1}{x} \\
y' &= x^{\sin x}(\cos x \cdot \ln x + \frac{1}{x} \cdot \sin(x)).
\end{aligned}$$

Problem 5. Use linear approximation to find the approximate value of $\sqrt{390}$.

SOLUTION. Let $f(x) = \sqrt{x}$, $a = 400$, $x = 390$. Then $f(a) = 20$ and $f'(a) = \frac{1}{2\sqrt{400}} = \frac{1}{40}$. Then, using linear approximation, we obtain

$$\sqrt{390} \simeq 20 + \frac{1}{40} \cdot (-10) = 20 - 0.25 = 19.75.$$

Problem 6. (EXTRA CREDIT)

Show using the formal definition of derivative that $(\sin x)' = \cos x$

Solution:

$$\begin{aligned}
(\sin x)' &= \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0} \frac{\sin(x)\cos h + \cos(x)\sin(h) - \sin(x)}{h} = \\
&= \sin(x)\cdot \lim_{h\to 0}\frac{\cos h - 1}{h} + \cos(x)\cdot \lim_{h\to 0}\frac{\sin h}{h} = \\
&= \sin x \cdot 0 + \cos x \cdot 1 = \cos x.
\end{aligned}$$