P.270

{9, 13} — determine concavity.

P.287

2, 7, 16, 24, 25, 35, 37

P.296

1, 6, 12, 15

P.270

⑨ $Y = x^{-1}$

$Y' = -x^{-2}$

$Y'' = 2x^{-3} \neq 0.$

y'': $- - - - \;|\; + + + + +$

0

Y is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$.

⑬ $Y = (1+x)^{-2}$

$Y' = -2(1+x)^{-3}$

$Y'' = 6(1+x)^{-4} \neq 0.$

y'': $+ + + + + \;|\; + + + + +$

-1

Y is concave up on $(-\infty, -1) \cup (-1, \infty)$.

P.287

② $Y = x^{\frac{1}{2}}$

$Y' = \frac{1}{2}x^{-1/2} \neq 0.$

Y': $|\; + + + + + + + \;|$

0 4

Y is increasing on $[0, 4]$.

$(0,0)$ is a local + global minimum.

$(4,2)$ is a local and global maximum.

⑦ $Y' = 3(x-1)^2 = 0$ when $x = 1$.

Y': $+ + + + + \;|\; + + + + +$

1

Y is increasing on $(-\infty, 1] \cup [1, \infty) = \mathbb{R}$.

There are no local or global extrema.

(16) $y' = \frac{1}{2}(1+x^2)^{-1/2}(2x)$

$= x(1+x^2)^{-1/2} = 0$ when $x=0$.

y': $- - - - -$ | $+ + + + +$

0

y is increasing on $(0,\infty)$
and decreasing on $(-\infty,0)$.
$(0,1)$ is a local and global min.

(24) $f(x) = \ln x + x^{-1}$. $(x>0)$.

$f'(x) = \frac{1}{x} - x^{-2}$

$f''(x) = -x^{-2} + 2x^{-3} = 0$

$\Rightarrow -x+2 = 0 \Rightarrow x = 2$.

f'': | $+ + + + +$ | $- - - - -$

0 2

f is concave up on $(0,2)$, concave down on $(2,\infty)$.

So $\boxed{(2, \ln 2 + \frac{1}{2}) \text{ is the only inflection point.}}$

(25) $f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''(0) = 0$. ✓✓✓

f'': $+ + + +$ | $+ + + +$

0

f'' does not change sign at $x=0$. ✓✓✓

So f does not have an inflection point at $x=0$. ✓✓✓

(35) a) degree of top = degree of bottom, and ratio of leading coefficients is one.

Thus $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$.

b) As $x \to 1^-$, top approaches 1, bottom approaches 0, and the quotient is negative (as x approaches 1 from the left)

So $\lim_{x\to 1^-} f(x) = -\infty$

(Continued on next page.)

#35 continued

As $x \to 1^+$, the top approaches 1, the bottom approaches 0, and the quotient is positive.

So $\lim_{x \to 1^+} f(x) = \infty$.

c) $f(x) = \frac{x}{x-1}$ $(x \neq 1)$

$$f'(x) = \frac{(x-1)\cdot 1 - x \cdot 1}{(x-1)^2} = \frac{-1}{(x-1)^2} \neq 0.$$

f': – – – – – | – – – – (1)

f is decreasing on $(-\infty, 1)$ and $(1, \infty)$.

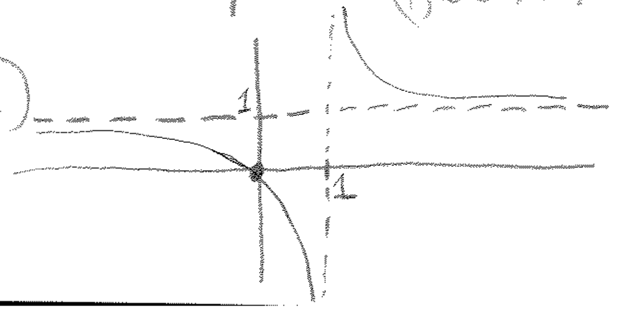
f has no local extrema.

d) $f''(x) = 2(x-1)^{-3} \neq 0$.

f'': – – – – – | + + + + + + (1)

f is concave down on $(-\infty, 1)$ and concave up on $(1, \infty)$. No inflection points.

e)



37 a) $f(x) = \frac{2x^2 - 5}{x+2}$, $x \neq -2$.

top: $2(-2)^2 - 5 = 3 \neq 0$.

bottom: $(-2) + 2 = 0$.

bottom is 0 but top is not 0 when $x = -2$, so $x = -2$ is a vertical asymptote.

b) $f'(x) =$

$$\frac{(x+2)(4x) - (2x^2 - 5)\cdot 1}{(x+2)^2}$$

$$= \frac{4x^2 + 8x - 2x^2 + 5}{(x+2)^2}$$

$$= \frac{2x^2 + 8x + 5}{(x+2)^2} = 0$$

$$\Rightarrow 2x^2 + 8x + 5 = 0$$

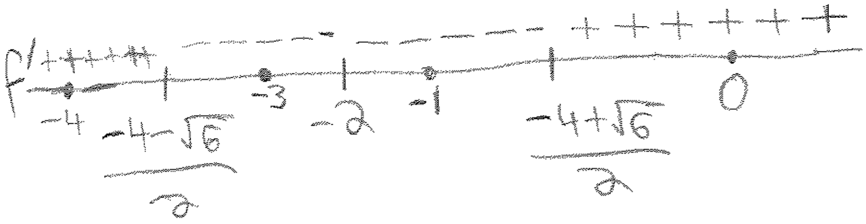
$$\Rightarrow x = \frac{-8 \pm \sqrt{24}}{4}$$

$$= \frac{-8 \pm 2\sqrt{6}}{4}$$

$$= \frac{-4 \pm \sqrt{6}}{2}.$$

(Next Page)

#37 Continued



f is increasing on $\left(-\infty, \frac{-4-\sqrt{6}}{2}\right)$,

decreasing on $\left(\frac{-4-\sqrt{6}}{2}, -2\right)$,

decreasing on $\left(-2, \frac{-4+\sqrt{6}}{2}\right)$,

increasing on $\left(\frac{-4+\sqrt{6}}{2}, \infty\right)$.

f has a local maximum at $x = \frac{-4-\sqrt{6}}{2}$,

and a local minimum at $x = \frac{-4+\sqrt{6}}{2}$

d) $$x+2 \overline{\smash{)}\,2x^2+0x-5}$$ quotient $2x-4$

$$\begin{array}{r} 2x^2+0x-5 \\ 2x^2+4x \\ \hline -4x-5 \\ -4x-8 \\ \hline 3 \end{array}$$

$$2x^2-5 = (x+2)(2x-4)+3$$

$$\frac{2x^2-5}{x+2} = 2x-4+\frac{3}{x+2}$$

$\boxed{2x-4 \text{ is an oblique asymptote.}}$

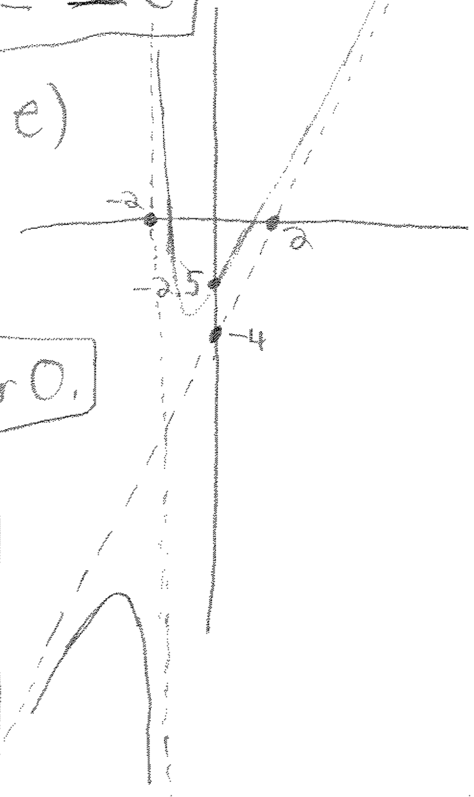
c) $$f''(x) = \frac{(x+2)^2(4x+8)-(2x^2+8x+5)\cdot 2(x+2)}{(x+2)^4} = 0$$

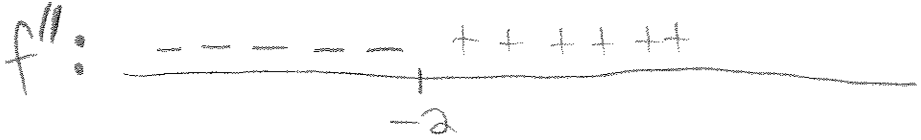
$$\Rightarrow 2(x+2)^2 - 2x^2 - 8x - 5 = 0$$

$$\Rightarrow \cancel{2x^2} + \cancel{8x} + 8 - \cancel{2x^2} - \cancel{8x} - 5 = 0$$

$\Rightarrow 3 = 0$ ... impossible. So $f''(x)$ is never 0.

e)





f is concave down on $(-\infty, -2)$

and concave up on $(-2, \infty)$.

$\boxed{\text{No inflection points.}}$

P. 296

①

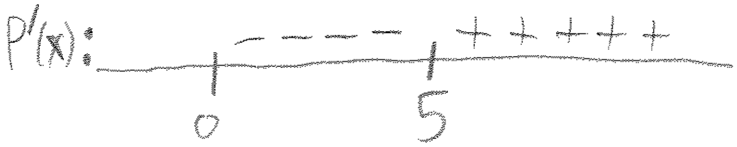
$xy = 25$

$y = \frac{25}{x}$

$P(x) = 2x + 2\left(\frac{25}{x}\right) = 2x + 50x^{-1}, \ x > 0.$

$P'(x) = 2 - 50x^{-2} = 0$

$\Rightarrow 2 = \frac{50}{x^2} \Rightarrow x^2 = 25 \Rightarrow x = 5.$



$x = 5$ is a global min for $P(x)$.

$P(5) = 2 \cdot 5 + \frac{50}{5} = \boxed{20 \text{ cm.}}$

⑥

$x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \sqrt{16 - x^2}.$

$A(x) = \frac{x}{2}\sqrt{16 - x^2}.$

Let's maximize

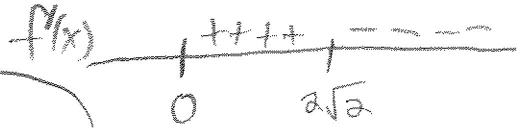
$A(x)^2 = \frac{x^2(16 - x^2)}{4}$

$= \frac{16x^2}{4} - \frac{x^4}{4} = 4x^2 - \frac{x^4}{4} = f(x).$

$f'(x) = 8x - x^3 = 0$

$\Rightarrow 8 - x^2 = 0$

$\Rightarrow x^2 = 8 \Rightarrow x = \sqrt{8} = 2\sqrt{2}$



$f(x)$ has a global max at $x = 2\sqrt{2}$.

$A(2\sqrt{2})$

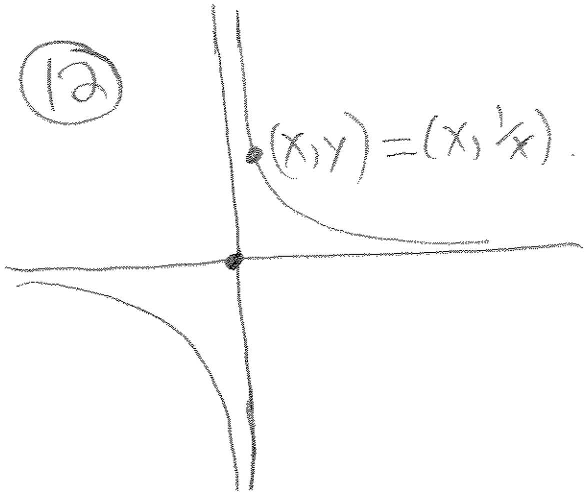
$= \frac{2\sqrt{2}}{2}\sqrt{16 - 8}$

$= \sqrt{2}\sqrt{8} = \sqrt{2}\sqrt{2}\sqrt{4}$

$= 2 \cdot 2 = 4.$

$\boxed{4 \text{ cm}^2}$

⑫



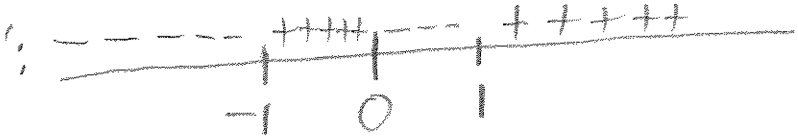
$$D(x)=\sqrt{x^2+\left(\tfrac{1}{x}\right)^2}$$

$$\Rightarrow f(x)=D(x)^2=x^2+x^{-2}.$$

$$f'(x)=2x-2x^{-3}=0$$

$$\Rightarrow x=\frac{1}{x^3}$$

$$\Rightarrow x^4=1 \Rightarrow x=\pm 1.$$



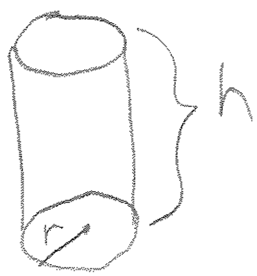
f(x) has local minima at x = 1 and x = -1.

$f(1)=2$. $f(-1)=2$.

So f has global minima at x=1 and x=-1.

$D(1)=D(-1)=\sqrt{2}$ is the smallest possible distance from a point on the curve to the origin.

⑮



$$\pi r^2 h=1000$$

$$\Rightarrow h=\frac{1000}{\pi r^2}$$

$$A(r)=2\pi r^2+2\pi r h$$

$$=2\pi r^2+2\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$=2\pi r^2+2000r^{-1}.$$

$$A'(r)=4\pi r-2000r^{-2}=0$$

$$\Rightarrow \pi r=\frac{500}{r^2}$$

$$\Rightarrow r^3=\frac{500}{\pi}$$

$$\Rightarrow \boxed{r=\sqrt[3]{\frac{500}{\pi}}}.$$

$$\boxed{h=\frac{1000}{\pi\left(\frac{500}{\pi}\right)^{2/3}}}.$$