4.4.5 p. 210 (76, 78, 85)

76 $h(s) = \frac{1}{s^2+2} = (s^2+2)^{-1}$.

$h'(s) = -(s^2+2)^{-2}(2s) = (-2s)(s^2+2)^{-2}$.

$h''(s) = (-2s)(-2(s^2+2)^{-3}(2s)) - 2(s^2+2)^{-2}$.

78 $f(x) = x^{-2} + x - 1$.

$f'(x) = -2x^{-3} + 1$.

$f''(x) = 6x^{-4}$.

85 $p(x) = ax^2 + bx + c$.

$p(0) = c = 3$.

$p'(x) = 2ax + b$. $p'(0) = b = 2$.

$p''(x) = 2a$. $p''(0) = 2a = 6$, so $a = 3$.

Thus $\boxed{p(x) = 3x^2 + 2x + 3.}$

4.5, p. 215 (4, 18, 40, 58)

4) $f'(x) = -\cos x - \sin x$.

18) $f(x) = \left(\cos(x^2-1)\right)^2$.

$f'(x) = 2\left(\cos(x^2-1)\right)^1\left(-\sin(x^2-1)(2x)\right)$

$= \boxed{-4x\cos(x^2-1)\sin(x^2-1)}$.

40) $f(x) = \frac{1}{\cos x} \cdot \cos x = 1$.

So $f'(x) = \boxed{0}$.

58) $f'(x) = \dfrac{(1-x^2)\left(-\csc(3-x^2)\cot(3-x^2)(-2x)\right) - \csc(3-x^2)(-2x)}{(1-x^2)^2}$

$= \boxed{\dfrac{2x\csc(3-x^2)\left[(1-x^2)\cot(3-x^2)+1\right]}{(1-x^2)^2}}$

3

4.6.1, p.221 (4, 10, 20, 40, 54)

4 $f'(x) = 3e^{(2-5x)}(-5) = \boxed{-15e^{(2-5x)}}$

10 $f'(x) = \boxed{(2x)(3e^{3x}) + 2e^{3x}}.$

20
$$f'(x) = e^{\cos(1-2x^3)}\left(-\sin(1-2x^3)(-6x^2)\right)$$
$$= \boxed{6x^2 e^{\cos(1-2x^3)} \sin(1-2x^3)}.$$

40 $f'(x) = \boxed{3^{(x^3-1)}(\ln 3)(3x^2)}.$

54 See ex. 4, p. 220.

$$\lim_{h\to 0} \frac{e^{5h}-1}{3h} = \lim_{h\to 0} \frac{e^{5h}-1}{5h} \cdot \frac{5h}{3h}$$

$$= \lim_{h\to 0} \frac{e^{5h}-1}{5h} \cdot \lim_{h\to 0} \frac{5h}{3h} = 1 \cdot \frac{5}{3} = \boxed{\frac{5}{3}}.$$

4.7.4, p. 233 (2, 5, 9, 23–30, 63, 64)

2 If $y = f^{-1}(x)$, then $f(y) = x$,

so $\sqrt{y-1} = x \Rightarrow y - 1 = x^2 \Rightarrow y = x^2 + 1$

$\Rightarrow f^{-1}(x) = x^2 + 1$.

So $(f^{-1})'(x) = 2x$.

Also, $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$ (☆)

$$= \frac{1}{\frac{1}{2}\left((x^2+1)-1\right)^{-1/2}} = \frac{1}{\frac{1}{2}\left(x^2\right)^{-1/2}}$$

$$= \frac{1}{\frac{1}{2}x^{-1}} = \frac{1}{\frac{1}{2}\cdot\frac{1}{x}} = \frac{1}{\left(\frac{1}{2x}\right)} = 2x,$$

So the seemingly weird formula (☆) actually gives the right answer.

5 $y = f^{-1}(x) \Rightarrow f(y) = x$

$\Rightarrow 3 - 2y^3 = x \Rightarrow 3 - x = 2y^3$

$\Rightarrow \frac{3-x}{2} = y^3 \Rightarrow y = \left(\frac{3-x}{2}\right)^{1/3} = f^{-1}(x).$

So $(f^{-1})'(x) = \frac{1}{3}\left(\frac{3-x}{2}\right)^{-2/3}\left(\frac{-1}{2}\right) = \left(\frac{-1}{6}\right)\left(\frac{3-x}{2}\right)^{-2/3}.$

But also, $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$ (★)

$= \frac{1}{-6\left(\left(\frac{3-x}{2}\right)^{1/3}\right)^2} = \frac{1}{-6\left(\frac{3-x}{2}\right)^{2/3}}$

$= \left(\frac{-1}{6}\right)\left(\frac{3-x}{2}\right)^{-2/3}.$

So (★) gives the right answer.

9 $(f^{-1})'(2) = \dfrac{1}{f'(f^{-1}(2))} = \dfrac{1}{f'(3)}$.

$f'(x) = \frac{1}{2}(x+1)^{-1/2} \Rightarrow f'(3) = \frac{1}{2}(4)^{-1/2} = \frac{1}{4}$.

So $(f^{-1})'(2) = \dfrac{1}{(1/4)} = \boxed{4.}$

23 $f'(x) = 1/(x+1)$

24 $f'(x) = \dfrac{1}{3x} \cdot 3 = \boxed{\dfrac{1}{x}}$.

25 $f'(x) = \dfrac{1}{1-2x}(-2) = \boxed{\dfrac{-2}{1-2x}}$.

26 $f'(x) = \dfrac{1}{3+4x}(4) = \boxed{\dfrac{4}{3+4x}}$.

27 $f'(x) = \dfrac{1}{x^2}(2x) = \boxed{\dfrac{2}{x}}$.

28 $f'(x) = \dfrac{1}{1-x^2}(-2x) = \boxed{\dfrac{-2x}{1-x^2}}$.

29 $f'(x) = \frac{1}{x^3}(3x^2) = \boxed{\frac{3}{x}}$.

30 $f'(x) = \frac{1}{x^3-1}(3x^2) = \boxed{\frac{3x^2}{x^3-1}}$.

63 $y = x^x$

$\ln y = \ln(x^x) = x \ln x$.

$\frac{1}{y} y' = \ln x + x\left(\frac{1}{x}\right) = \ln x + 1$

$\Rightarrow y' = y(\ln x + 1) = \boxed{x^x(\ln x + 1).}$

64 $y = x^{2x}$. $\ln y = \ln(x^{2x}) = 2x \ln x$.

$\frac{1}{y} y' = 2 \ln x + 2x\left(\frac{1}{x}\right) = 2\ln x + 2$

$\Rightarrow y' = y(2\ln x + 2) = \boxed{x^{2x}(2\ln x + 2).}$