HW #4 solutions

4.14

13)

a)

$$y' = \lim_{h \to 0} \frac{5(-1+h)^2 - 5}{h}$$

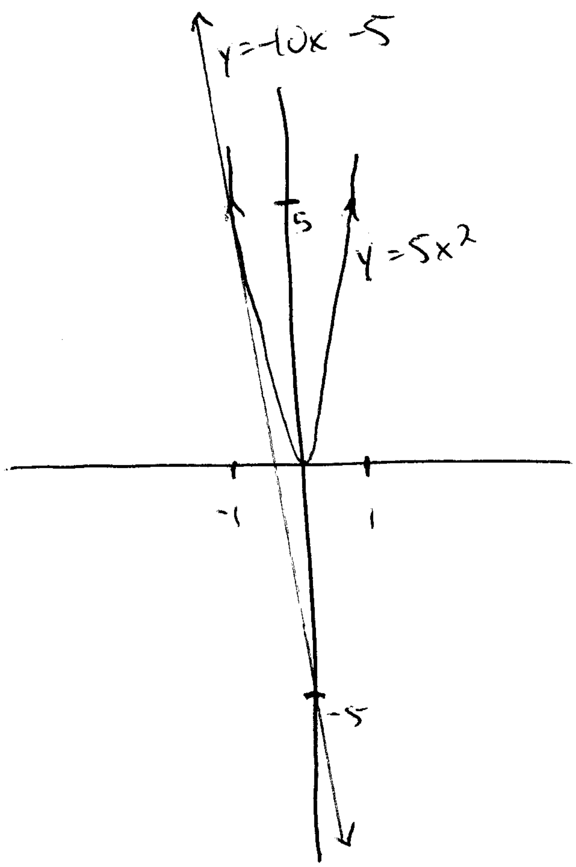
$$= \lim_{h \to 0} \frac{5 - 10h + 5h^2 - 5}{h}$$

$$= \lim_{h \to 0} (-10 + 5h) = -10$$

b) Equation of tangent line:

$(y-5) = -10(x+1)$, or $y = -10x - 5$

c)



14)

a) $$y' = \lim_{h\to 0} \frac{-2(1+h)^2 + 2}{h}$$

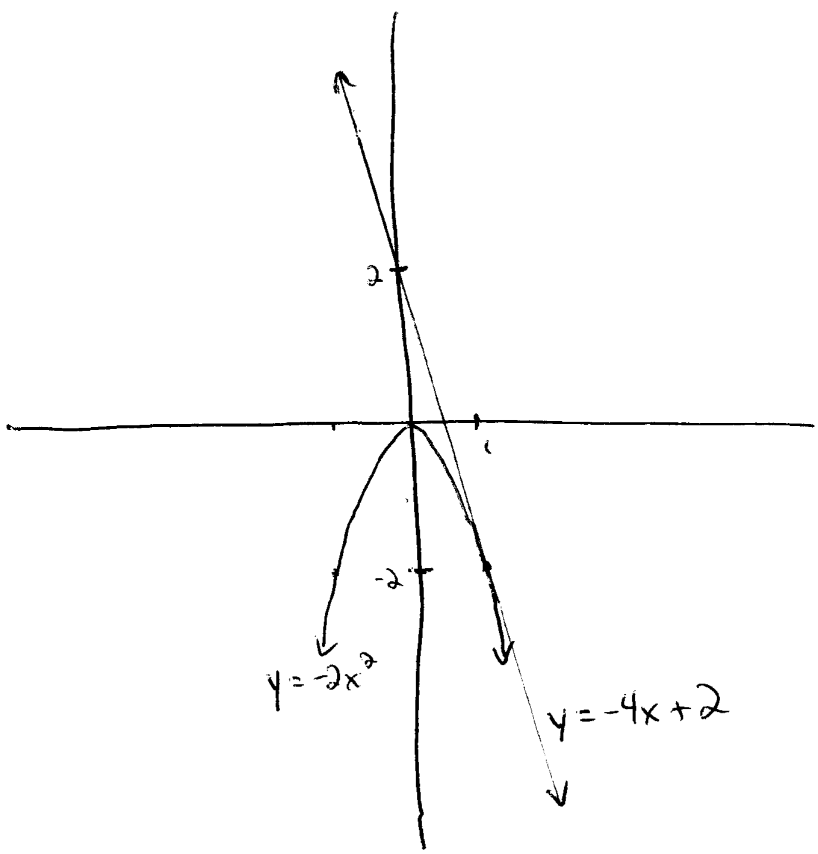
$$= \lim_{h\to 0} \frac{\cancel{-2} - 4h - 2h^2 \cancel{+2}}{h}$$

$$= \lim_{h\to 0} (-4 - 2h) = -4$$

b) Equation of tangent line.

$(y+2) = -4(x-1)$, or $y = -4x + 2$

c)



17)

$$y = \sqrt{x}$$

$$y' = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x}+h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

27)

$$\lim_{h \to 0} \frac{2(a+h)^2 - 2a^2}{h} = f'(a)$$

where $f(x) = 2x^2$.

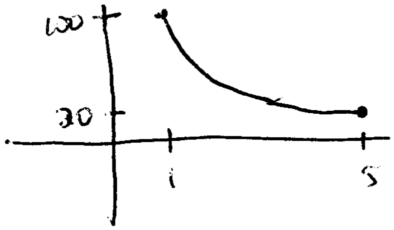
29)

$$\lim_{h \to 0} \frac{\frac{1}{(2+h)^2+1} - \frac{1}{5}}{h} = f'(a)$$

where $f(x) = \frac{1}{x^2+1}$ and $a = 2$

32) $s(t) = \frac{100}{t}, \quad 1 \le t \le 5$.

a)



b) Avg velocity $= \frac{\Delta y}{\Delta x} = \frac{20 - 100}{5 - 1} = \frac{-80}{4} = -20$. km/hr

c)
$$s'(2) = \lim_{h \to 0} \frac{\frac{100}{2+h} - \frac{100}{2}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{\cancel{200 - 200} - 100h}{2(2+h)}}{h} =$$
$$= \lim_{h \to 0} \frac{-100}{2(2+h)} = -25 \text{ km/hr}$$

41)
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

Setting $\frac{dN}{dt} = 0$ we get $N=0$ or $1 - \frac{N}{K} = 0$
i.e. $N=0$ or $N=K$. As $N=0$ is the trivial solution corresponding to population size zero, we take $N=K$ as the equilibrium population size.