TOPICS FOR INDIVIDUAL PROJECTS (MATH 191)

1. **The enhanced linking number.**
   The *linking number* is an invariant of links which is useful for detection of linking and distinguishing links in many cases. However, in some cases, even though the linking seems to be non-trivial, this invariant is equal to 0. The *enhanced linking number* is a more powerful invariant of a similar type.
   The project involves learning the definition, main properties and basic applications of the enhanced linking number, as well as doing some computations of this invariant.
   MAIN REFERENCE: C. Livingston, *"Enhanced linking number"*, AMM, v. 110, no. 5, p. 361-380.
2. **The theory of hitches.**
   A *hitch* is a mode of wrapping a rope around a post so that, with the help of some friction, the rope holds to the post. A mathematical analysis of the properties of hitches is very interesting and is closely related to the theory of knots and links.
   MAIN REFERENCE: B. F. Bayman, *"The theory of hitches"*, Amer. J. Physics, vol. 45, no. 2 (1977), p. 185-190.
3. **Braids, Artin groups and the Jones polynomials.**
   During the lectures, we will introduce the Jones polynomial via the bracket polynomial. The goal of this project is to examine the close relation of the Jones polynomial with braids and representations of the Artin groups (which is much closer to the Jones' original approach).
   INITIAL REFERENCE: section 1.7 of L. Kauffman *"Knots and Physics"*.
4. **Knots, Abstract Tensors and the Yang-Baxter equation.**
   Learn how knot or link diagrams can be interpreted as abstract tensor diagrams. Study the diagrammatic approach to Yang-Baxter equation and its solutions.
   INITIAL REFERENCE: section 1.8 of L. Kauffman *"Knots and Physics"*.
5. **Solutions to the Yang-Baxter equation, Hopf algebras and link invariants.**
   This project is closely related to no. 5, and may be considered as its continuation. You will need to learn about bialgebras, Hopf algebras and the Faddeev-Reshetikhin-Takhtajan formalizm. An extensive background reading is required. To start off, use the
   INITIAL REFERENCE: section 1.10 of L. Kauffman *"Knots and Physics"*.
6. **The knot quandle.**
   Quandles are algebraic objects closely related to groups and algebras. It turns out that one can naturally associate a quandle to a knot. This gives a new invariant for knots.
   MAIN REFERENCE: D. Joyce *"A classifying invariant of knots, the knot quandle"*, J. Pure Appl. Alg., v. 23, pp. 37-65.
7. **Seifert Surfaces of Knots.**
   Seifert surface of a knot is a connected orientable surface bounded by the

given knot. Knots with the operation of connected sum behave similarly to integers with the operation of multiplication: there is a unit (the unknot), the primes, and even the unique factorization theorem.
STARTING REFERENCE: from the book by D. Lickorish *"An introduction to Knot theory"*.

8. **Knots and fundamental groups.**
   Learn about the fundamental group of the complement of a knot (a very powerful invariant related to topology).
   SOURCE: almost any knot theory book.
9. **Obtaining 3-manifolds by surgery.**
   Every closed connected orientable 3-manifold can be obtained by "surgery" on S^3.
   SOURCE: e.g., R. Lickorish *"An introduction to Knot Theory"*.
10. **Generalizations of the Jones polynomial.**
    INITIAL REFERENCE: R. Lickorish *"An introduction to Knot Theory", Ch. 15.*
11. **Knots and topology of the DNA.**
    INITIAL REFERENCE: Sumners, De Witt, *"Lifting the curtain: using topology to probe the hidden action of enzymes"*, Notices AMS (42), 5, 1995.