A correction of Today's lecture.

We want to establish a relation between

$$V(\ ) \quad \text{and} \quad V(\ ).$$

In other words, we want to understand how annihilation (represented by) and crossing ($\longrightarrow$) interact.

So far, we assume only

$$V = A \cdot V + B \cdot V$$

$$V = A' \cdot V + B' \cdot V$$

$$V_{(K \sqcup \bigcirc)} = \delta \cdot V_K$$

and conditions $AA' = 1$, $BB' = 1$, $\delta = -\left(\frac{A}{B} + \frac{B}{A}\right) = -\left(\frac{A'}{B'} + \frac{B'}{A'}\right)$

which follow from channeling and cross-channeling.

Now, the computation:

$$V = A' \cdot V + B' \cdot V = A' \cdot \left(A' V + B' \underbrace{V}_{= V}\right) +$$

$$+ B' \cdot \left(A' \underbrace{V}_{= \delta \cdot V} + B' \underbrace{V}_{= V}\right) =$$

$$= \left((A')^2 + (B')^2 + \delta A' B' \cancel{+ A'B'}\right) V + A'B'$$

Now, the formula for $\delta = -\left(A'/B' + B'/A'\right)$ implies that the first three terms add up to 0.

So, $V = A'B' \cdot V$

Lecture 9.

$$V = A \cdot V + B \cdot V = (A\delta + B) V$$

$$A\delta + B = A \cdot \left(-\frac{A}{B} - \frac{B}{A}\right) + B = -\frac{A^2}{B}.$$

Thus, $$V = V \iff \boxed{B = -A^2}$$

Need: $\boxed{B = -A^2}$

Let $A = -\sqrt{t}$, $B = -t$

Then

$$V = -\sqrt{t} \cdot V - t\,V \qquad \Big|\ \text{multiply by } t^{-1}$$

$$V = -\frac{1}{\sqrt{t}} V - \frac{1}{t} V \qquad \Big|\ \text{multiply by } t:$$

$$\Rightarrow \quad t^{-1} V - t\,V = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right) V.$$

Therefore, $V_K(t)$ is the (1-variable) Jones polynomial.