1/

Lecture 8.

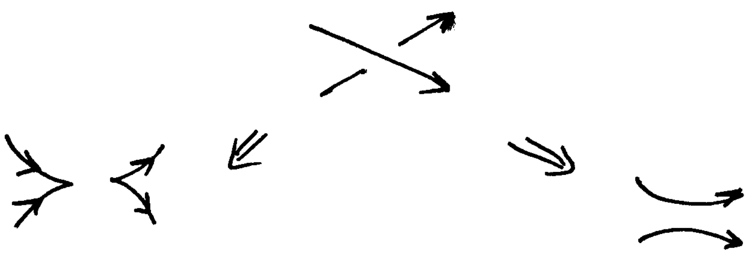
(After the talk of S. Reed on "Knot quandles").

An oriented state model for $V_K(t)$.

Recall: $V_K(t) = (-t^{3/4})^{w(K)} \cdot \langle K \rangle (t^{-1/4})$.

Splicings of unoriented diagrams:

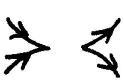
Splicings of oriented diagrams:



not in the category
of link diagrams
(because of the kinks)

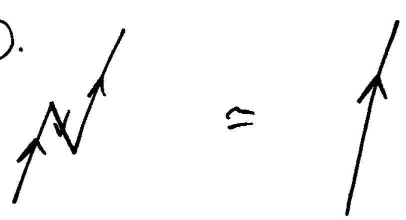
— time.

 - represents an interaction (a pass-by)

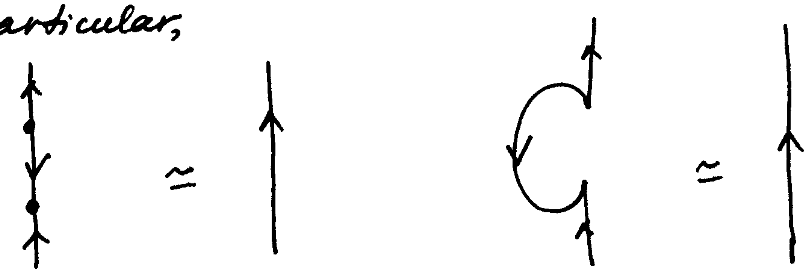
 - creation and annihilation

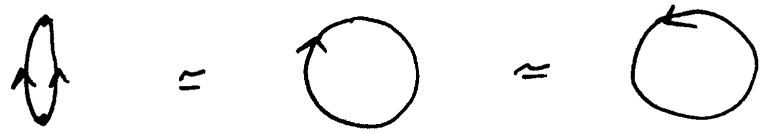
In order to get topological invariance, need to have rules on cancellation of creation with annihilation:

[1] ①.

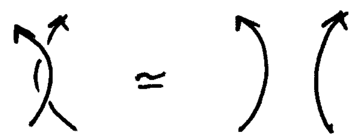


In particular,

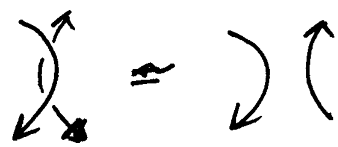




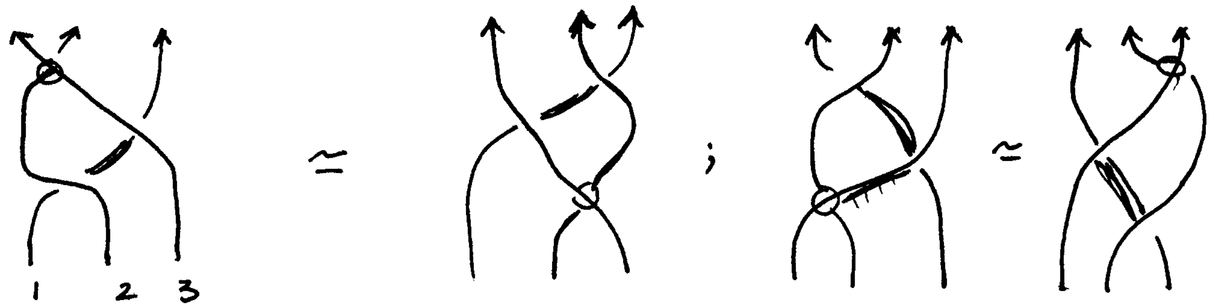
2 <u>Channel property</u>:



3 <u>Cross-channel property</u>:



4. <u>Triangle invariance</u>:



Analogously to the bracket, let

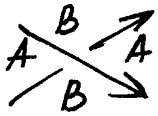
$$V = A \cdot V + B \cdot V$$

$$V = A' \cdot V + B' \cdot V$$

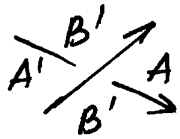
A, B, A', B' are weights to be determined from properties 2–4 above.

Assume also that $V_{(K \cup \bigcirc)} = \delta \cdot V_K$, where δ is a parameter.

3/



,



Using ② - ④ to set conditions on A, A', B, B':

② Unitarity (channeling)

$$V\left(\ \right) = A \cdot V + B \cdot V =$$

$$= A \cdot \left(A' V + B' V \right) +$$

$$+ B \cdot \left(A' \cdot V + B' \cdot V \right)$$

Thus,

$$\boxed{V = AA' \cdot V + (AB' + BB'\delta + BA') \cdot V}$$

Unitarity requires:

$$\boxed{\begin{aligned} AA' &= 1 \\ \delta &= -\left(\frac{A}{B} + \frac{A'}{B'}\right) \end{aligned}}$$

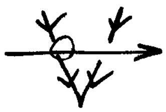
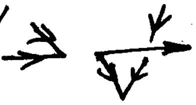
③ Similarly, cross-unitarity requires

$$\boxed{\begin{aligned} BB' &= 1 \\ \delta &= -\left(\frac{B'}{A'} + \frac{B}{A}\right) \end{aligned}}$$

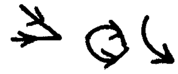
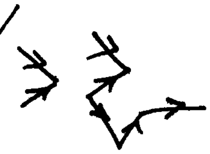
4/ Combining the two, we get:

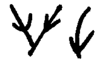
$$\boxed{A' = \frac{1}{A}, \quad B' = \frac{1}{B} \qquad \delta = -\left(\frac{A}{B} + \frac{B}{A}\right).}$$

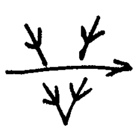
④. Annihilation and crossing:

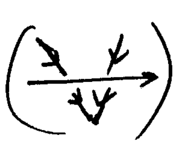
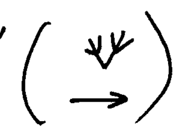
V  $= A' \cdot$  $+ B' \cdot V$  $=$

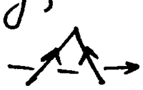
$= A' \cdot \Big(A' \cdot V$  $+ B' \cdot V$  $\Big) +$

$+ B' \Big(A' \cdot V$  $+ B' \cdot V$  $\Big) =$

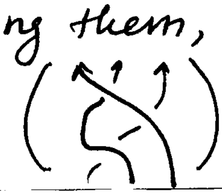
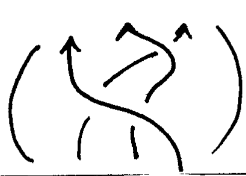
$= \left((A')^2 + (B')^2 + \delta_{A'B'} \right) V$  $+ A' \cdot B' \cdot$ 

V  $= A' \cdot B' \cdot V$  $= \frac{1}{AB} V$ 

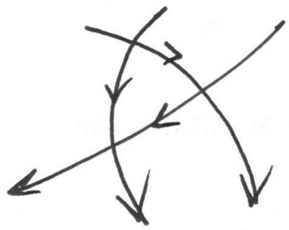
$V\Big($  $\Big) = \frac{1}{AB} V \Big($  $\Big)$

Similarly, can get relations for other triangle moves (such as   , etc.)

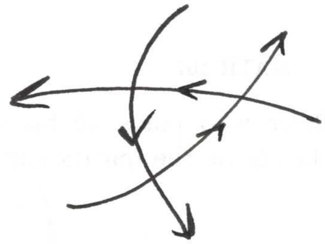
Substituting them, verify

$V\Big($  $\Big) = V\Big($  $\Big)$ and similar relations.

Notice that there are two types of triangles:

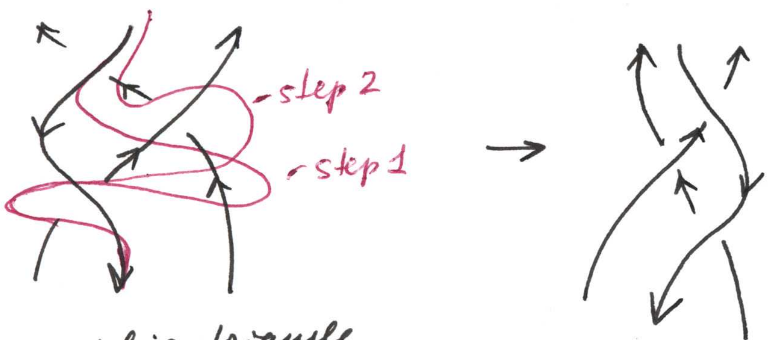


(central triangle doesn't have cyclic orientation)



(cyclic orientation of the central triangle)

These triangles can be reduced to each other using channel & cross-channel unitarity :



cyclic triangle in the center $\Rightarrow$ non-cyclic triangle.

Thus, the model

V  $= A\,V$  $+ B$ 

V  $= \frac{1}{A} V$  $+ \frac{1}{B} V$ 

is invariant under regular isotopy for oriented link diagrams.

What about the moves of type similar to R1?