

HOMEWORK 5

Problem 1. Using only the three axioms of the quandle (see Hand-out for the lecture by Steven Read), prove the following relations in a quandle:

$$\begin{aligned} x \triangleleft x &= x, \\ x \triangleright (x \triangleleft y) &= y, \\ z \triangleleft (y \triangleleft x) &= (z \triangleleft y) \triangleleft (z \triangleleft x), \end{aligned}$$

where $x, y, z \in Q$ are arbitrary elements in a quandle Q .

Problem 2. Compute the knot quandle for the figure eight knot (4_1).

Problem 3. Recall that one can associate a group with a quandle in the following way. Let Q be a quandle with elements x, y, z, \dots . The elements of the resulting group G are the equivalence classes $\bar{x}, \bar{y}, \bar{z}$, where the equivalence relation is such that

$$\overline{x \triangleright y} = \bar{x} \cdot \bar{y} \cdot \bar{x}^{-1}.$$

With this definition describe the group arising from the knot quandle of the trefoil.

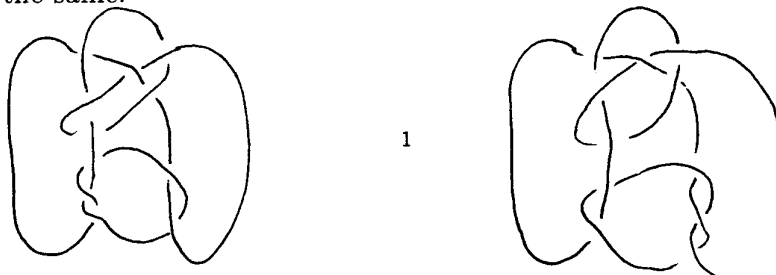
Problem 4. Prove that in the oriented state model of the Jones polynomial the following relations are satisfied:

$$\begin{aligned} V_{\text{cross}} &= t\sqrt{t} V_{\text{smooth}} \\ V_{\text{smooth}} &= \frac{1}{-t\sqrt{t}} V_{\text{cross}} \end{aligned}$$

Problem 5. Prove that the Jones polynomials of the two links below coincide:



Problem 6. Check that the Jones polynomial of the two knots below are the same:



Hint for #5, #6: What is $V_{\text{Jones}}(L_1 \# L_2)$, where L_1, L_2 - links; $\#$ - connected sum (joining compon