HOMEWORK 3

Problem 1. Take an arbitrary regular knot projection. Create a knot diagram out of it in the following way: starting from a chosen point, alternate the types of crossings (over, under, over, ...). Why does this never lead to a contradiction? (*Hint:* consider shadings of the diagram with two colors (black and white), so that any to adjacent regions have different colors. Understand first why such a shading is always possible).

Problem 2. What is the first non-alternating knot in the table of knots?

Problem 3. Let W be the Whitehead link. Compute the bracket polynomial $\langle W \rangle$ and show that W is indeed linked.

Problem 4. For $n \geq 1$, let K_n be the torus link of type $(2, n)$. (See Homework 1 for a definition). Find a recursive formula for computing the bracket polynomials of torus links, $\langle K_n \rangle$. Compute $\langle K_n \rangle$ for $n \leq 4$. Show that K_n is not ambient isotopic to its mirror image for any n.

Problem 5. Let L be a link and U be the unknot. Compute the Jones polynomial of $L \cup U$ (express it in terms of the Jones polynomial of L).

Problem 6. Compute the Jones polynomial of the figure-eight knot (4_1):

1. First, use the definition of the Jones polynomial via the bracket polynomial.
2. Use the skein relation of the Jones polynomial.

From your answer, conclude that 4_1 is ambient isotopic to its mirror image.

Problem 7. Does the Jones polynomial of a knot depend on its orientation? What about links?

Problem 8. Let L_+, L_- and L_0 be three links which differ at just one crossing (as in the skein relation for the Jones polynomial). Suppose that L_+ has n components. How many components can L_- and L_0have? Show that if a link has an odd number of components, then its Jones polynomial contains only integer powers of t (i.e., $\dots$, $t^{-1}, 1, t, \dots$). Show that if a link has an even number of components, only half-integer powers $(\dots, t^{-1/2}, t^{1/2}, t^{3/2}, \dots)$ appear.