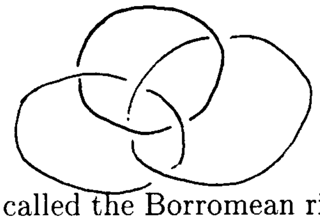
HOMEWORK 2

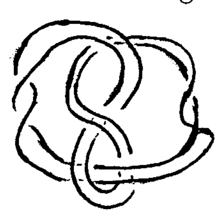
Problem 1. The following 3-component link



is called the Borromean rings. (It has an interesting property that if you remove a component, the remaining two rings are no loner linked). Choose an orientation for each component and compute the linking number of Borromean rings. Does the answer depend on the choice of orientation? (Note: for a link with more then two components, the linking number is half the sum of crossing signs of all crossings between different components of the link).

Problem 2. Show that any diagram of a link can be changed into a diagram of several copies of the unlink by suitable crossing changes. (By a crossing change, we mean the change of an overcrossing to an undercrossing, and vice versa). What is the effect of a crossing change on the linking number (consider all possibilities)? Prove that the linking number is always an integer (even though it is defined as one-half of a sum of integers).

Problem 3. Compute the linking number of the following ~~knot:~~ link



Problem 4. Compute the 3-colouring invariant $\tau(5_1)$. Does this invariant distinguishes it from the unknot?

Problem 5. Compute the 3-coloring invariant $\tau(6_3)$.

Problem 6. A knot invariant i is called *subadditive* if it satisfies $i(K_1 \# K_2) \leq i(K_1)+i(K_2)$, where $\#$ denotes the connected sum of knots. Prove that both the unknotting number and the crossing number are subadditive invariants. (It is still unknown whether the strict equality holds for both of this invariants).

Problem 7. Show that the two loops

can be eliminated with Redemeister moves R2 and R3.

Problem 8. Show that if a link diagram has a p-colouring, then it can be coloured starting with any prescribed color (e.g., color 0) on any prescribed arc.

Problem 9. Prove that any link with more then 1 component is 2-colourable. Prove that no knot is 2-colourable.