Class 285I, Sprin 2008: Hilbert bimodules and subfactors of finite index
Instructor: Sorin Popa
Meetings: M 2-4, W 2-4 in MS6627.

We begin by reviewing the notion of Hilbert modules over type II_1 factors, and more generally over finite von Neumann (vN) algebras. Then we consider *Hilbert bimodules*, as a generalized notion of symmetries for II_1 factors and vN algebras, and introduce an important tool for studying such objects, called the *the basic construction*. We'll discuss at length several applications to this construction, such as:

• A non-commutative treatement of *relative compactness* and *relative weak mixing* for actions of groups on finite vN algebras.

• Providing several equivalent definitions of finite index for subalgebras;

• Proving criteria for unitary conjugacy between subalgebras in a factor (*intertwining subalgebras* techniques).

• Defining relative amenability, relative property (T), relative Haagerup property, etc.

We then prove Jones' theorem showing that the dimension of Hilbert bimodules can only take values in the set $\{4cos^2\pi/n \mid n \geq 3\} \cup [0, \infty]$. We introduce the *standard invariant* of a subfactor, and prove several axiomatizations of these objects as "abstract" objects. Finally, we will explain how subfactor techniques can be used to approach problems in *non-commutative ergodic* theory, i.e., in the study and classification of actions of groups on arbitrary (not necessarily abelian) finite vN algebras.

All registered students in the class who will attend regularly will get A, but they are expected to present some assigned material in the 290 Student Seminar, Mo 4-5, in MS 6229.

ATTENTION: The class will start Wednesday April 2'nd, and instead of 2-4pm, we will have it 4-6pm that day, in room 6627. (There will be no Functional Analysis Seminar that day.)