Suppose $A$ is a $2 \times 2$ matrix and that $v_1$ is an eigenvector of $A$ with eigenvalue 2 and $v_2$ is an eigenvector of $A$ with eigenvalue $-2$.

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

(a) Find the general solution to $y' = Ay$

$$c_1 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

(b) Draw a picture of all solutions to $y' = Ay$. Make sure to include the solutions that always stay in some eigenspace of $A$.

The solutions in green are solutions of the form $ce^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. They always stay in the eigenspace $E_2$ of $A$ and as $t \to \infty$ they rush away from the origin.

The solutions in red are solutions of the form $ce^{-2t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. They always stay in the eigenspace $E_{-2}$ of $A$ and as $t \to \infty$ they get closer to the origin.

The solutions in black are linear combinations of the green and red solutions. As $t \to \infty$ the green component gets larger and the red component gets smaller until the green component dominates.