MATH 54 SUMMER 2017, QUIZ 23

Find the projection of $\sin(x)$ on the subspace $\text{span}\{1, x\}$ in the inner product space $C([0, \pi])$ with the inner product given below.

$$\langle f, g \rangle = \int_0^\pi f(x)g(x) \, dx$$

Warning: with the inner product above, $\{1, x\}$ is not an orthogonal set!

[Hint: $\int_0^\pi \sin(x) \, dx = 2$ and $\int_0^\pi x \sin(x) \, dx = \pi$.]

1. Use Gram-Schmidt to find an orthogonal basis for $\text{span}\{1, x\}$

$$f_1 = 1$$

$$f_2 = x - \text{proj}_{\text{span}\{1, x\}}(x) = x - \frac{\langle x, 1 \rangle}{\langle 1, 1 \rangle} 1 = x - \frac{\pi^2/2}{\pi} 1 = x - \frac{\pi}{2}$$

$$\langle 1, 1 \rangle = \int_0^\pi 1 \, dx = \pi$$

$$\langle x, 1 \rangle = \int_0^\pi x \, dx = \left[ \frac{x^2}{2} \right]_0^\pi = \frac{\pi^2}{2}$$

Orthogonal basis: $\{1, x - \frac{\pi}{2}\}$

2. Find the projection of $\sin(x)$ using this orthogonal basis

$$\text{proj}_{\text{span}\{1, x\}}(\sin(x)) = \text{proj}_{f_1}(\sin(x)) + \text{proj}_{f_2}(\sin(x))$$

$$\langle \sin(x), 1 \rangle = \int_0^\pi \sin(x) \, dx = 2$$

$$\langle \sin(x), x - \frac{\pi}{2} \rangle = \left[ \frac{x \sin(x)}{2} \right]_0^\pi = \pi - \frac{\pi}{2} \cdot 2$$

$$\langle \sin(x), 1 \rangle = \frac{2}{\pi} 1 + \frac{\pi}{\pi} \left( x - \frac{\pi}{2} \right) \left( x - \frac{\pi}{2} \right)$$

$$\langle \sin(x), x - \frac{\pi}{2} \rangle = \left( \frac{2}{\pi} \right) \frac{\pi}{\pi}$$

$$\text{proj}_{f_2}(\sin(x)) = \frac{2}{\pi}$$

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