MATH 54 SUMMER 2017, QUIZ 11

Mark each of the following true or false. You do not have to provide an explanation.

(a) The set of invertible $3 \times 3$ matrices is a subspace of the vector space of $3 \times 3$ matrices.

False. The zero matrix, $\begin{bmatrix}0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}$ is not invertible.

(b) The set of constant functions from $\mathbb{R}$ to $\mathbb{R}$ is a subspace of the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ (i.e. $C(\mathbb{R})$). (A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is constant if for all real numbers $x$ and $y$, $f(x) = f(y)$.)

True. 1. The zero function (i.e. $x \mapsto 0$) is constant.
   2. If $f$ and $g$ are constant then for all $x, y \in \mathbb{R}$,
      
      $$(f+g)(x) = f(x) + g(x) = f(y) + g(y) = (f+g)(y).$$
      
      So $f+g$ is constant.
   3. If $f$ is constant and $c \in \mathbb{R}$ then for all $x, y \in \mathbb{R}$,
      $$(cf)(x) = cf(x).$$
      
      So $cf$ is constant.

(c) The set of polynomials with integer coefficients of degree at most 3 is a subspace of the vector space of polynomials with real coefficients of degree at most 3 (i.e. $\mathbb{P}_3$).

False. It is not closed under scalar multiplication.
For instance $x^3 + x$ has integer coefficients, but
\[ \frac{1}{2} (x^3 + x) = \frac{1}{2} x^3 + \frac{1}{2} x \] does not.

(d) The following vectors in $M_{2\times2}$ span all of $M_{2\times2}$ (recall that $M_{2\times2}$ is the vector space of all $2 \times 2$ matrices).

\[
\begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\]

True. If $\begin{bmatrix}a & b \\ c & d\end{bmatrix}$ is any $2 \times 2$ matrix then

\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
= a \begin{bmatrix}1 & 0 \\ 0 & 0\end{bmatrix} + b \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix} + c \begin{bmatrix}0 & 0 \\ 1 & 0\end{bmatrix} + d \begin{bmatrix}0 & 0 \\ 0 & 1\end{bmatrix}
\]

so $\begin{bmatrix}a & b \\ c & d\end{bmatrix}$ is spanned by the set of matrices $\{
\begin{bmatrix}1 & 0 \\ 0 & 0\end{bmatrix},
\begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix},
\begin{bmatrix}0 & 0 \\ 1 & 0\end{bmatrix},
\begin{bmatrix}0 & 0 \\ 0 & 1\end{bmatrix}\}$

(e) The following vectors in $\mathbb{P}_4$ are linearly independent: $x + 1$, $x^4 + x$, and $x^4 - 1$.

False.

\[ (x^4 + x) - 1 \cdot (x + 1) - 1 \cdot (x^4 - 1) = 0 \]

Date: July 5, 2017.