Review

1. For $A$ shown below, find:

   \[
   A = \begin{bmatrix}
   1 & -2 & 0 & 3 \\
   0 & 1 & 1 & -1 \\
   0 & 0 & 0 & 0 
   \end{bmatrix}
   \]

   (a) a vector in $\text{Col } A$
   (b) a vector not in $\text{Col } A$
   (c) a vector in $\text{Null } A$
   (d) a vector not in $\text{Null } A$

2. If $A$ is an $n \times m$ matrix then how many solutions does $A\mathbf{x} = \mathbf{b}$ have if:

   (a) $\text{Null } A = \{0\}$ and $\mathbf{b} \in \text{Col } A$?
   (b) $\text{Null } A \neq \{0\}$ and $\mathbf{b} \in \text{Col } A$?
   (c) $\mathbf{b} \notin \text{Col } A$?

3. If $A$ is an $n \times n$ invertible matrix, what are $\text{Null } A$ and $\text{Col } A$?

Bases, Dimension, and Rank

1. Is \( \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\} \) a basis for $\mathbb{R}^2$?

2. Can 2 vectors ever form a basis for $\mathbb{R}^3$? Can 4 vectors ever form a basis for $\mathbb{R}^3$? What are the possible sizes of a basis for $\mathbb{R}^3$?

3. Suppose $V$ is a subspace of $\mathbb{R}^n$. What is the largest possible size of a basis for $V$? What if you know that $V \neq \mathbb{R}^n$?

4. Find a basis for $\text{Col } A$ and a basis for $\text{Null } A$.

   \[
   A = \begin{bmatrix}
   1 & 2 & 0 & 4 \\
   2 & 4 & 5 & -3 \\
   5 & 10 & 0 & 20 
   \end{bmatrix}
   \]

5. For $A$ as in the previous problem, what is rank $A$? What is $\text{dim}(\text{Null } A)$?

6. Suppose $A$ is a $5 \times 7$ matrix of rank 3. What is $\text{dim}(\text{Null } A)$? (Hint: think about pivots.)

7. If $A$ is an $n \times m$ matrix with linearly independent columns, what is the rank of $A$?
Vector Spaces

Which of the following are vector spaces?

1. The set of convergent sequences of real numbers whose limit is 0.
2. The set of functions from $\mathbb{Z}$ to $\mathbb{Z}$.
3. The set of differentiable functions $f : \mathbb{R} \to \mathbb{R}$ such that $\frac{d}{dx} f(x) = f(x)$.
4. The set of polynomials with real coefficients of degree exactly 3.
5. The set of matrices in REF.