Differential Equations

1. Which of the following functions are solutions to the differential equation $y'' - y = 2 - t^2$?
   (a) $f: \mathbb{R} \to \mathbb{R}$ defined by $f(t) = t^2$
   (b) $g: \mathbb{R} \to \mathbb{R}$ defined by $g(t) = te^t$
   (c) $h: \mathbb{R} \to \mathbb{R}$ defined by $h(t) = t^2 + e^t$

2. Which of the functions in problem 1 are solutions to the initial value problem
   
   \[
   y'' - y = 2 - t^2
   \]
   \[
   y(0) = 1
   \]
   \[
   y'(0) = 1
   \]

3. Find the general solution of the following differential equations.
   (a) $y'' - 2y' - 3y = 0$
   (b) $y'' - 5y' = 0$
   (c) $y''' + 5y'' + 4y' = 0$

4. Find a differential equation for which $e^{7t} + 4e^{-3t}$ is a solution.

5. Find the solution to each of the following initial value problems.
   (a) $y'' + y' = 0$, $y(0) = 2$, $y'(0) = 1$
   (b) $y''' + 5y'' + 4y' = 0$, $y(0) = 8$, $y'(0) = -9$, $y''(0) = 33$

Definitions and Theorems

Definitions:
- Differential Equation, ODE
- Solution to a differential equation
- Initial value problem (IVP)
- Linear, constant coefficient, homogeneous ODE
- Auxiliary equation

Most important idea today: Finding the general solution to a linear, constant coefficient, homogeneous ODE just means finding the kernel of some linear transformation.