Complex Eigenvalues

For this section, let

\[
A = \begin{bmatrix}
3/2 & -5/2 \\
5/2 & -3/2
\end{bmatrix}
\]

1. Find the eigenvalues of \(A\) and for each eigenvalue, find a corresponding eigenvector.

2. Find real numbers \(a\) and \(b\) such that \(A\) is similar to

\[
\begin{bmatrix}
a & -b \\
b & a
\end{bmatrix}
\]

3. Find \(r\) and \(\theta\) such that \(A\) is similar to

\[
r \begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\]

4. Calculate and draw \(e_1\), \(Ae_1\), \(A^2e_1\), \(A^3e_1\), and \(A^4e_1\). How do the vectors you calculated and drew relate to the values for \(r\) and \(\theta\) you found above?

Dot Product

1. \(u = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}\), \(v = \begin{bmatrix} 0 \\ -6 \\ 4 \end{bmatrix}\), \(w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\)

   (a) Find the lengths of \(u\) and \(v\)—i.e. find \(||u||\) and \(||v||\).
   (b) Find the distance between \(u\) and \(v\)—i.e. find \(\text{dist}(u, v)\).
   (c) Find \(u \cdot v\) and \(u \cdot w\).
   (d) Find the cosine of the angle between \(u\) and \(v\) and the cosine of the angle between \(u\) and \(w\).
   (e) Find a unit vector in the same direction as \(u\).

2. If \(u\) and \(v\) are vectors in \(\mathbb{R}^n\) and the angle between them is \(\pi/2\) radians (90°), what is \(u \cdot v\)?

3. If \(u\) is any vector in \(\mathbb{R}^n\), what is \(u \cdot 0\)?

4. Is any pair of vectors from problem (1) orthogonal?

5. Show that for all vectors \(v\) in \(\mathbb{R}^n\), \(v \cdot v \geq 0\). When is it equal to 0?

6. Suppose \(W\) is a subspace of \(\mathbb{R}^n\). Let \(H\) be the set of all vectors that are orthogonal to every vector in \(W\). Formally,

\[
H = \{ v \in \mathbb{R}^n \mid \text{for all } u \text{ in } \mathbb{R}^n, v \cdot u = 0 \}.
\]

Is \(H\) a subspace of \(\mathbb{R}^n\)?

7. What is the orthogonal complement of \(\{0\}\)? (Assume this means the zero vector in \(\mathbb{R}^n\).)