After finishing the quiz, work on the first four questions.

**Diagonalization**

1. What is $A^{100}$?
   
   $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

2. If $A = PBP^{-1}$ then what is $A^{100}$?

3. (a) Suppose $v_1$ and $v_2$ are eigenvectors of a matrix $A$ with corresponding eigenvalues 5 and $-3$. What is $A^{100}v_1$? What about $A^{100}v_2$?
   
   (b) If $v_3 = 2v_1 + 6v_2$, what is $A^{100}v_3$?

4. 
   
   $$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \quad B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

   (a) What is $[e_1]_B$? What is $[e_2]_B$?
   
   (b) What are $A^{100}e_1$ and $A^{100}e_2$? (Hint: the vectors in $B$ are eigenvectors of $A$.)
   
   (c) What is $A^{100}$? (Hint: Use the previous part.)

5. What is the maximum number of eigenvalues a $5 \times 5$ matrix can have?

6. Try to diagonalize the following matrices (not all of them are necessarily diagonalizable.)

   (a) $\begin{bmatrix} 2 & 2 \\ 0 & 3 \end{bmatrix}$
   
   (b) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$
   
   (c) $\begin{bmatrix} 1 & 2 \\ -3 & -6 \end{bmatrix}$
   
   (d) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$

7. Find a matrix $A$ such that $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are eigenvectors of $A$ with corresponding eigenvalues $-5$ and 1.

8. For each statement below, explain why it is true or provide a counterexample to show it is false.

   (a) Every invertible matrix is diagonalizable.
   
   (b) Every diagonalizable matrix is invertible.
   
   (c) If $A$ is a nonzero matrix and $A^2 = 0$ then $A$ is not diagonalizable.
   
   (d) Every $2 \times 2$ matrix with more than one eigenvalue is diagonalizable.
   
   (e) Every upper triangular matrix is diagonalizable.