Matrix Algebra

1. Suppose that A and B are 2×2 matrices such that $A\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\5\end{bmatrix}$ and $B\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}-2\\-3\end{bmatrix}$. Find a solution to $(A+B)\mathbf{x} = \begin{bmatrix}-1\\2\end{bmatrix}$.
2. Suppose that A and B are 2×2 matrices such that $A\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ and B is invertible, with inverse $B^{-1} = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$. Find a nontrivial solution to $(AB)\mathbf{x} = \mathbf{0}$.
3. What is I_n^{-1}?
4. Find a 2×2 matrix A such that A is nonzero but $A^2 = 0$.
5. What is the determinant of the following matrix?

$$\begin{bmatrix}1 & 7 & 8 & 1 & 2 & 3\\2 & -9 & 81 & 2 & 7 & 0\\3 & 4 & 7 & 3 & 7 & -1\\4 & 1 & 1 & 4 & 1 & 1\\5 & 7 & -3 & 5 & 13 & 788\\6 & -1 & -2 & 6 & -4 & -5\end{bmatrix}$$

Challenge problem: Find a formula for $\begin{bmatrix}3 & 1\\0 & 3\end{bmatrix}^n$.