Matrices ①

a) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 2\cdot(-2) + 3\cdot 4 \\ -1\cdot 1 + 0\cdot(-2) + 1\cdot 4 \end{bmatrix} = \begin{bmatrix} 1-4+12 \\ -1+0+4 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ Not defined.

c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$ Not defined.

d) $\begin{bmatrix} 0 & 7 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0\cdot 4 + 7\cdot 3 + (-1)\cdot 2 + 2\cdot 1 \end{bmatrix} = \begin{bmatrix} 21 \end{bmatrix}$

e) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0\cdot 1 + 0\cdot 2 + 0\cdot 3 \\ 0\cdot 1 + 0\cdot 2 + 0\cdot 3 \\ 0\cdot 1 + 0\cdot 2 + 0\cdot 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

f) $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 2\cdot 0 + 3\cdot 0 \\ 6\cdot 0 + 5\cdot 0 + 4\cdot 0 \\ 7\cdot 0 + 8\cdot 0 + 9\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Linear Independence

① Show that each list of vectors is linearly dependent.

a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

$$3 \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

not all 0

b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 17 \\ -3 \end{bmatrix}$

$$0 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1 \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0 \cdot \begin{bmatrix} 17 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

not all 0 (it's okay that some are 0)

c) $\vec{u}, \vec{v}, 3\vec{u} - 4\vec{v}$ where $\vec{u}, \vec{v} \in \mathbb{R}^4$

$$(-3) \cdot \vec{u} + 4 \cdot \vec{v} + 1 \cdot (3\vec{u} - 4\vec{v}) = \vec{0}$$

not all 0

(2) Check whether each list of vectors is linearly independent

a) $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$ $\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$ RREF

pivot in every column ⇒ linearly independent

b) $\begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}3\\1\end{bmatrix}$ $\left[\begin{array}{cc|c}2&3&0\\0&1&0\end{array}\right]$ REF

pivot in every column ⇒ linearly independent

c) $\begin{bmatrix}3\\1\\-2\end{bmatrix}, \begin{bmatrix}5\\0\\1\end{bmatrix}, \begin{bmatrix}2\\-1\\3\end{bmatrix}$

$\left[\begin{array}{ccc|c}3&5&2&0\\1&0&-1&0\\-2&1&3&0\end{array}\right]$ $R_3 = 3R_3 + 2R_1$ $\left[\begin{array}{ccc|c}3&5&2&0\\1&0&-1&0\\0&13&13&0\end{array}\right]$ $R_2 = 3R_2 - R_1$ $\left[\begin{array}{ccc|c}3&5&2&0\\0&-5&-5&0\\0&13&13&0\end{array}\right]$

$R_3 = 5R_3 + 13R_2$ $\left[\begin{array}{ccc|c}3&5&2&0\\0&-5&-5&0\\0&0&0&0\end{array}\right]$ REF

↳ free variable ⇒ not linearly independent

③ Give an example of

a) Vectors in $\mathbb{R}^2$ which are linearly dependent and span all of $\mathbb{R}^2$

Ex. 1: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Ex. 2: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) Vectors in $\mathbb{R}^3$ which are linearly independent but don't span all of $\mathbb{R}^3$

Ex 1: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Ex 2: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$