Vector Arithmetic and Span

1. Is it possible to add together multiples of $\mathbf{a}$ and $\mathbf{b}$ to get $\mathbf{c}$?

   (a) $\qquad \mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (b) $\qquad \mathbf{a} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

2. Draw $\text{span}\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$, $\text{span}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$, and $\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

3. If $\mathbf{u}$ and $\mathbf{v}$ are both in $\text{span}\{\mathbf{w}_1, \mathbf{w}_2\}$ then is $5\mathbf{u} - 2\mathbf{v}$ also in the span of $\mathbf{w}_1$ and $\mathbf{w}_2$?

4. (a) Is it possible to find two vectors in $\mathbb{R}^2$ that *don't* span all of $\mathbb{R}^2$?

   (b) Is is possible to find two vectors in $\mathbb{R}^2$ whose span does not include $\mathbf{0}$?

   (c) Is it possible to find two vectors in $\mathbb{R}^3$ whose span is all of $\mathbb{R}^3$?

Challenge problem: What is $\text{span}\{\mathbf{u}_1, \mathbf{u}_2\} \cap \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$?

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$