Systems of First Order Linear ODEs

1. Check if each function given below is a solution to $\mathbf{y}'(t) = A(t)$.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

(a)
$$\mathbf{y}_1(t) = \begin{bmatrix} -e^{3t} \\ e^{3t} \\ 4e^{3t} \end{bmatrix}$$

(b)
$$\mathbf{y}_2(t) = \begin{bmatrix} \sin(t) \\ 2 \\ 3e^{5t} \end{bmatrix}$$

2. Suppose $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with eigenvalue 5. Show that $\mathbf{y}(t) = \begin{bmatrix} e^{5t} \\ 2e^{5t} \end{bmatrix}$ is a solution to the differential equation $\mathbf{y}'(t) = A\mathbf{y}(t)$.

3. Find the general solution to the following ODE.

$$\mathbf{y}'(t) = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix} \mathbf{y}(t)$$

4. Find the solution to the following initial value problem

$$\mathbf{y}'(t) = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix} \mathbf{y}(t)$$
$$\mathbf{y}(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

5. What is the long term behavior of the solution you found in the previous problem? I.e. when t is large, what does $\mathbf{y}(t)$ look like—what is its norm and approximately what direction is it pointing in?