Review $\quad V = \text{span}\{1, x, x^2\} = \mathbb{P}_2$

$T : V \to V \qquad T(f) = 5\frac{d^2 f}{dx^2} + 2\frac{df}{dx}$

Pick a basis for V and write the matrix for T relative to that basis

$B = \{1, x, x^2\}$

$T(1) = 0 \rightsquigarrow [0]_B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$T(x) = 5\frac{d^2x}{dx^2} + 2\frac{dx}{dx} = 2 \rightsquigarrow 2\cdot 1 + 0\cdot x + 0\cdot x^2 \quad \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

$T(x^2) = 5\frac{d^2x^2}{dx^2} + 2\frac{dx^2}{dx} = 5\cdot 2 + 2\cdot(2x) = 10 + 4x$

$= 10\cdot 1 + 4\cdot x + 0\cdot x^2 \quad \begin{bmatrix} 10 \\ 4 \\ 0 \end{bmatrix}$

$_B[T]_B = \begin{bmatrix} 0 & 2 & 10 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

① Suppose $y_1(t)$ and $y_2(t)$ are both sol'ns to

$$ay'' + by' + cy = 0$$

Show that $c_1 y_1(t) + c_2 y_2(t)$ is also a sol'n (where $c_1, c_2 \in \mathbb{R}$)

$$a(c_1 y_1(t) + c_2 y_2(t))'' + b(c_1 y_1(t) + c_2 y_2(t))'$$
$$+ c(c_1 y_1(t) + c_2 y_2(t))$$

$$= a \cdot c_1 \cdot y_1'' + a \cdot c_2 \cdot y_2'' + b \cdot c_1 y_1' + b \cdot c_2 y_2'$$
$$+ c \cdot c_1 y_1 + c \cdot c_2 y_2$$

$$= c_1(ay_1'' + by_1' + cy_1) + c_2(ay_2'' + by_2' + cy_2)$$

$$= c_1 \cdot 0 + c_2 \cdot 0 = 0.$$

Finding a solution to $ay''+by'+cy=0$ $=$ Find an element of the kernel of $T: C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ defined by $T(f) = a\frac{d^2f}{dt^2} + b\frac{df}{dt} + cf$

Previous exercise: Since T is a linear transformation, its kernel is a subspace — i.e. closed under addition and scalar multiplication

② Suppose e^{2t} and e^{3t} are both sol'ns to

$$ay'' + by' + cy = 0$$

and $\sin(t)$ is a solution to

$$ay'' + by' + cy = -5\cos(t) + 5\sin(t)$$

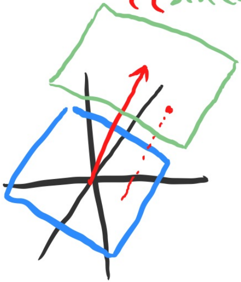
Find all solutions to $ay'' + by' + cy = -5\cos(t) + 5\sin(t)$

$\sin(t) + c_1 e^{2t} + c_2 e^{3t}$ $\qquad$ $T = a\frac{d^2}{dt^2} + b\frac{d}{dt} + c \cdot I$

$$T(\sin(t) + c_1 e^{2t} + c_2 e^{3t}) = T(\sin(t)) + c_1 T(e^{2t}) + c_2 T(e^{3t})$$

$$= (-5\cos(t) + 5\sin(t)) + c_1 \cdot 0 + c_2 \cdot 0$$

$$= -5\cos(t) + 5\sin(t)$$



③ Let $T: \underbrace{C^\infty(\mathbb{R})}_{} \longrightarrow C^\infty(\mathbb{R})$

↳ set of inf. differentiable functions

$$T(f) = 5\frac{d^2f}{dt^2} + 2\frac{df}{dt}$$

Find all eigenvectors of T

$T(f) = \lambda f$ for some $\lambda \in \mathbb{R}$

$5f'' + 2f' = \lambda f$

$5f'' + 2f' - \lambda f = 0$

$5r^2 + 2r - \lambda$

↑ char. eq'n

roots: $r = \dfrac{-2 \pm \sqrt{4 - 4 \cdot 5 \cdot (-\lambda)}}{10}$

$= \dfrac{-1 \pm \sqrt{1+5\lambda}}{5}$

$e^{\frac{-1 \pm \sqrt{1+5\lambda}}{5} t}$ is an eigenvector with eigenvalue λ

Solving Diff. Eq'ns

1. Learn algorithms to find solutions

2. Understanding where these algorithms came from & why they work ← this is where linear algebra comes in