① Which of the following functions are solutions to

$$y'' - y = 2 - t^2$$

a) $f(t) = t^2$ $\quad (t^2)'' - (t^2) = 2 - t^2$ Solution

b) $g(t) = e^t$ $\quad (e^t)'' - e^t = e^t - e^t = 0$ Not a solution

c) $h(t) = \sin(t) + t^2$ $\quad (\sin(t) + t^2)'' - (\sin(t) + t^2) =$ Not a sol'n

$-\sin(t) + 2 - \sin(t) - t^2 = -2\sin(t) + 2 - t^2$

d) $k(t) = 2e^t + t^2$ Solution

$(2e^t + t^2)'' - (2e^t + t^2) = 2e^t + 2 - 2e^t - t^2 = 2 - t^2$

② Which of the functions above are solutions to

$$y'' - y = 2 - t^2 \quad y(0) = 2 \quad y'(0) = 2$$

a) $f(0) = 0$ Not a solution

d) $k(0) = 2e^0 + 0^2 = 2$ ✓ Is a solution

$k'(0) = 2e^0 + 2 \cdot 0 = 2$

Solving Homogeneous Linear Ordinary Differential Equations

Given: $ay'' + by' + cy = 0$

① Char. polynomial: $a\lambda^2 + b\lambda + c$

② roots of char. polynomial: r_1, r_2

③ $e^{r_1 t}$ and $e^{r_2 t}$ are sol'ns and so are all linear combinations of them

④ If $r_1 = r_2$, $te^{r_1 t}$ is also a solution

⑤ General solution:

$$C_1 e^{r_1 t} + C_2 e^{r_2 t} \quad \text{if} \quad r_1 \neq r_2$$

$$C_1 e^{r_1 t} + C_2 t e^{r_1 t} \quad \text{if} \quad r_1 = r_2$$

C_1, C_2 are any real numbers

④ Find the general sol'n to each differential equation

a) $y'' - 2y' - 3y = 0$

$\lambda^2 - 2\lambda - 3 = (\lambda - 3)(\lambda + 1)$

roots: $3, -1$

$c_1 e^{3t} + c_2 e^{-t}$

Check: $(e^{3t})'' - 2(e^{3t})' - 3e^{3t}$

$= 9e^{3t} - 6e^{3t} - 3e^{3t} = 0$

b) $y''' + 5y'' + 4y' = 0$

$\lambda^3 + 5\lambda^2 + 4\lambda = \lambda(\lambda^2 + 5\lambda + 4)$

$= \lambda(\lambda + 4)(\lambda + 1)$

roots: $0, -4, -1$

$c_1 e^{0t} + c_2 e^{-4t} + c_3 e^{-t} = c_1 + c_2 e^{-4t} + c_3 e^{-t}$

c) $y'' - 6y' + 9y = 0$

$\lambda^2 - 6\lambda + 9 = (\lambda - 3)(\lambda - 3)$

root: 3 (w/ mult. 2)

$c_1 e^{3t} + c_2 t e^{3t}$

Check: $(te^{3t})'' - 6(te^{3t})' + 9te^{3t}$

$= \cancel{9te^{3t}} - 6(\cancel{e^{3t}} + \cancel{3te^{3t}})$

$+ (\cancel{3e^{3t}} + \cancel{3e^{3t}} + \cancel{9te^{3t}}) = 0$

d) $y''' - 5y'' = 0$

$\lambda^3 - 5\lambda^2 = \lambda^2(\lambda - 5)$

roots: 5, 0 (w/ mult. 2)

$c_1 e^{5t} + c_2 e^{0t} + c_3 t e^{0t} = c_1 e^{5t} + c_2 + c_3 t$

r is a root w/ mult. n

$e^{rt}, te^{rt}, t^2 e^{rt}, \ldots t^{n-1} e^{rt}$ sol'ns

5 Find differential eq'ns with solutions

a) $e^{7t}+4e^{-3t}$

$$(\lambda-7)(\lambda+3) = \lambda^2-4\lambda-21$$

$$\Downarrow$$

$$y''-4y'-21y=0$$

$$y''-4y'-21y=0$$

$$(e^{7t}+4e^{-3t})'' - 4(e^{7t}+4e^{-3t})' - 21(e^{7t}+4e^{-3t})$$

$$= \cancel{49e^{7t}} + \cancel{36e^{-3t}} - \cancel{28e^{7t}} + \cancel{48e^{-3t}} - \cancel{21e^{7t}} - \cancel{84e^{-3t}}$$

$$= 0$$

b) te^{2t}

$$(\lambda-2)^2 = \lambda^2-4\lambda+4$$

$$\Downarrow$$

$$y''-4y'+4y=0$$