Linear Ordinary Differential Equations

1. Which of the following functions are solutions to the differential equation $y'' - y = 2 - t^2$?
   
   (a) $f(t) = t^2$
   (b) $g(t) = e^t$
   (c) $h(t) = \sin(t) + t^2$
   (d) $k(t) = 2e^t + t^2$

2. Which of the functions in the previous problem are solutions to the initial value problem $y'' - y = 2 - t^2$, $y(0) = 1$, $y'(0) = 1$?

3. Show that if $f$ and $g$ are both solutions to the differential equation $y''' - 5y'' + 17y' - 3y = 0$ then so is $5f + 3g$.

4. Find the general solution to the following differential equations.

   (a) $y'' - 2y' - 3y = 0$
   (b) $y''' + 5y'' + 4y' = 0$
   (c) $y'' - 6y' + 9y = 0$
   (d) $y''' - 5y'' = 0$

5. For each function below, find a homogeneous linear ordinary differential equation to which it is a solution.

   (a) $e^{7t} + 4e^{-3t}$
   (b) $te^{2t}$