## Gram-Schmidt Algorithm

1. Find an orthogonal basis for the column space of the following matrix.

$$\begin{bmatrix} 1 & 3 & 10 \\ 2 & 5 & 4 \\ 3 & 5 & 8 \\ 1 & 2 & 3 \end{bmatrix}$$

2. What happens if you run the Gram-Schmidt algorithm with a set of vectors that is not linearly independent?

## Least Squares

1. Suppose the least squares solution to $A\mathbf{x} = \mathbf{b}$ is $\mathbf{v}$. What is $\text{proj}_{\text{Col}(A)}(\mathbf{b})$?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

2. Find a least squares solution to $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

## Orthogonal Complement

1. What is $\{\mathbf{0}\}^{\perp}$?
2. What is $(\mathbb{R}^n)^{\perp}$?
3. If $W$ is a subspace of $\mathbb{R}^n$, what is $W \cap W^{\perp}$?
4. If $W$ is a subspace of $\mathbb{R}^n$ and $\mathbf{x} \in W$, what are $\text{proj}_W(\mathbf{x})$ and $\text{proj}_{W^{\perp}}(\mathbf{x})$?
5. Show that $\text{Col}(A)^{\perp} = \text{Null}(A^T)$.

## Orthogonal Matrices

1. If $U$ is an orthogonal matrix, what is $U^T U$?
2. If $U$ is a square orthogonal matrix, what is $UU^T$? What if $U$ is not square?
3. If $U$ is an orthogonal $n \times m$ matrix and $\mathbf{x} \in \mathbb{R}^m$, show that $\|U\mathbf{x}\| = \|\mathbf{x}\|$.